AP Physics 1 and 2 Free-Response Question Bank

Phase 1 edition (cumulative)

Current course content in the 2015–2019 free-response style, with scoring guidelines

45 AP Physics 1 FRQs and 36 AP Physics 2 FRQs, sorted by topic

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Do not use these questions on tests. They are public, so students may have seen or memorized them. Use them for practice. To make similar questions for a test, use the prompt for Claude on the next pages.

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Directions for students. Show your work for each part in the space provided after that part. Answer exactly the question asked. Where a part asks you to explain or justify, an answer with no reasoning earns no credit, and a part that asks for a paragraph-length response is scored partly on whether the paragraph is logical and free of incorrect or irrelevant statements. Express algebraic answers in terms of the quantities named in the part and physical constants, as appropriate. Use $g = 9.8\ \mathrm{m/s^2}$ or $g = 10\ \mathrm{m/s^2}$ unless a question says otherwise.

Point values follow the 2015–2019 exams: AP Physics 1 used 7- and 12-point questions; AP Physics 2 used 10- and 12-point questions.

Writing your own questions with Claude

Questions in this bank are public, so do not use them on tests. To write new questions of similar quality, paste everything between BEGIN PROMPT and END PROMPT into Claude (claude.ai; turn on web search so it can read the College Board documents), then add one line at the end, for example: *Write one 12-point AP Physics 1 question on rotational dynamics, using DID-WHA? and WEE-Y/D.* Review what comes back before you use it. The problem-type templates and the explanation structure below are adapted from the author's handouts T.0.4 and T.1.

— BEGIN PROMPT (copy from here) —

ROLE AND FIRST STEPS. You are writing free-response questions (FRQs) for AP Physics 1 or AP Physics 2, with scoring guidelines, for a teacher. Before writing, read the College Board's current Course and Exam Description (CED) for the course and the released 2015–2019 FRQs and scoring guidelines. Use the CED for content and the 2015–2019 exams for style. Model the reformed-physics tradition that shaped these courses (Etkina's Investigative Science Learning Environment, Knight's College Physics, AMTA Modeling Instruction), not 1990s textbook problems (Giancoli, Cutnell and Johnson). If you cannot read the CED, say so and follow this specification.

LANGUAGE OF THE COURSE (use it exactly; never let it slip). (1) *Conserved* does not mean *constant*. A conserved quantity (energy, momentum, angular momentum, charge) is never created or destroyed; the quantity belonging to a system can change, but every change is accounted for by exchange with the surroundings across the system boundary. Energy of a system changes only by work done on it by external forces (and by heating); its momentum changes only by external impulse; its angular momentum changes only by external angular impulse. A quantity is *constant* for a system only when that exchange is zero. Never write "momentum is conserved, so it is constant" without naming the system and the reason the exchange is zero; never write "energy is not conserved" when you mean that the mechanical energy of a chosen system changed.

(2) Always name the system. Potential energy belongs to a system of interacting objects (the ball–Earth system), not to one object. Whether work done by gravity appears as work or as a change in potential energy depends on whether Earth is in the system.

(3) A force is an interaction: write "the force exerted on A by B". Free-body (force) diagrams show only forces exerted on the chosen object or system, each drawn from the object, with no net-force arrow and no "centripetal force" or "centrifugal force" as an extra force.

(4) Use representations as reasoning tools: force diagrams, energy bar charts (initial energy of the system, plus energy transferred in by work or heating, equals final energy) and momentum bar charts, motion diagrams, graphs, field and ray diagrams. Ask students to draw them and to use them.

(5) Reason about how a quantity changes (increases, decreases, stays the same, and why) before writing equations, and connect the words to the mathematics later.

EXAM STRUCTURE. AP Physics 1: 7-point (about 13 minutes) and 12-point (about 25 minutes) questions. AP Physics 2: 10-point (about 20 minutes) and 12-point (about 25 minutes) questions. Choose the size by the working time a prepared student needs, not by the number of rubric lines. A paragraph-length part ("In a clear, coherent, paragraph-length response that incorporates physics principles, explain . . . ") appears only in a 7-point AP Physics 1 question or a 10-point AP Physics 2 question, at most one per question; a 7-point question may be a single paragraph-length response (PLR). Longer questions use modest-length explanation parts instead. Combine about two templates (below) for a 7-point question and two or three for a 12-point question; repetition is allowed. Descriptions must be dense, full sentences that create real cognitive load.

PROBLEM-TYPE TEMPLATES.

BOLD (calculate or draw): draw a force diagram, energy bar chart, or graph; determine a quantity by writing fundamental relationships, substituting, and solving; or graph a derived expression.

DID-CABp (compare features of A and B): two similar situations A and B; "Is [feature] of A greater than, less than, or the same as [feature] of B? Briefly justify."

DID-CABy (find the feature that explains a difference): "Why is [feature] of A greater than that of B? Explain."

DID-EMS (does the equation make sense?): give a proposed equation; students test how each variable changes the result, check limiting cases, and compare with their own earlier reasoning.

DID-RIO (rank interrelated objects): several connected objects (for example circuit elements); rank by a quantity, using = for ties, and justify.

DID-WHA? (what will happen after . . . ?): "If [change], will [quantity] increase, decrease, or stay the same? Justify."

WEE-Y/D (without equations, explain why or determine): first explain in words how changing X affects Y; then derive Y algebraically; then identify the equation in the derivation that supports the verbal argument and explain why (not just the final expression).

WEE-HOW (explain how to calculate): instructions another student could follow, naming the principle used at each key step.

TOM-CR (critique reasoning): quote a student's claim and reasoning; students identify a correct feature and an incorrect feature with reasons, derive the relevant result, and show how their derivation expresses the correct feature and corrects the incorrect one.

TOM-VIOL (construct a violation): students draw a graph or data table that behaves plausibly in one regime but violates a named physics principle in another, and explain which feature violates it.

MXB (linearize data): give a data table; students choose quantities to graph that give a straight line, explain why (match their derived equation to y = mx + b), plot with a best-fit line, and get the desired value from the slope or intercept.

PEA-VAL (design an experiment to obtain a value): from listed equipment, a procedure another student could follow (what to measure, with which instrument, in what order), the analysis (often a linearized graph), which measurement's uncertainty matters more and why, and how a stated assumption failing would bias the result.

PEA-BOOLEAN (design an experiment to test a yes/no proposition): procedure, plus what the data would show if the answer were yes and what they would show if it were no.

PLR (paragraph-length response): one coherent argument, scored on reasoning components and on being logical and complete.

HOW AN EXPLANATION IS BUILT (what the scoring guideline should look for): Relationship (the principle, named or stated, for a named system); then what is the same in the two cases and why; then what is different and why; then so what (the conclusion that follows). Each link uses "because". For conservation laws, name the system, state what is shared or converted, and say why the exchange with the surroundings is zero or what it is.

SCORING RULES.

(1) A brief part with selection blanks (greater/less/same, yes/no, a ranking) and a brief explanation is worth exactly 1 point: no credit without the correct selection, and with it, the point requires a correct explanation, not a good-faith attempt.

(2) A paragraph-length part with selection blanks above it awards nothing for the blanks; its points are reasoning components plus a final point for a response that is logical, follows from start to finish, and contains no incorrect statements.

(3) Treat every response with a rebuttable presumption of ignorance: a point needs evidence that leaves no reasonable doubt the student knows and applied the idea. No points for vocabulary, a bare conclusion, or an equation written without saying what its terms mean in the situation. Make points "fat" (an integrated chain of reasoning), as on the 2015–2018 exams.

(4) Accept every valid method, and never require a relationship that is optional in the course (for example the area under an electric field–distance graph, or calculus); you may accept it.

(5) Give an example response for each explanation part, a note on how to score consistency with an earlier wrong answer, and at least one "trap": the most likely wrong idea, stated in a student's words.

DESIGN RULES.

(1) Before writing any wording, record: the governing relationship, the other relationships needed, the case that the question turns on (for example one snapshot versus two snapshots, on the verge of slipping, a system boundary that includes or excludes Earth), the object model (the physical setup), the representations, the templates, and the trap.

(2) No later part may give away an earlier part's answer, and no figure may give away an answer. Read the finished question as a student would to check.

(3) Data tables come from a model plus small realistic scatter, with stated uncertainties when students must judge consistency. Students may use g = 9.8 or 10 m/s^2.

(4) Check every number with code, including units and significant figures.

(5) Figures are grayscale and accurate; perspective drawings of three-dimensional objects must be geometrically correct.

PROCESS. Write the record from Design rule 1, then the question, then the scoring guideline. Then solve the question yourself as a strong student who has not seen the guideline, writing out every part, and compare part by part with the guideline. Fix every disagreement, ambiguity, give-away, or unfair point, and report what you changed.

— **END PROMPT** —

AP Physics 1: Contents

Each cell lists the questions in that chapter that use that format, with page numbers. A question can appear in more than one column, so the format totals add up to more than the number of questions. The Total column counts the questions in each chapter. Question 3.2 is the second question in chapter 3.

Chapter	Total	Calculate or draw	Compare two cases	Rank	Predict a change	Explain without equations	Critique a student	Does it make sense?	Paragraph response	Design an experiment	Analyze data
1 One-dimensional kinematics	3	1.1 (6), 1.2 (9)	1.1 (6), 1.2 (9), 1.3 (10)	–	–	1.1 (6), 1.2 (9)	–	1.1 (6)	1.2 (9), 1.3 (10)	–	–
2 Vectors and two-dimensional kinematics	2	2.2 (12)	2.2 (12)	–	2.1 (11)	–	–	2.2 (12)	2.1 (11)	–	–
3 Forces and Newton's laws	11	3.1 (13), 3.2 (14), 3.5 (18), 3.6 (20), 3.8 (25)	3.1 (13), 3.3 (16), 3.8 (25), 3.11 (29)	–	3.9 (27)	3.6 (20)	3.2 (14), 3.6 (20)	3.5 (18), 3.10 (28)	3.3 (16), 3.8 (25), 3.9 (27), 3.10 (28), 3.11 (29)	3.4 (17), 3.7 (23)	3.6 (20), 3.7 (23)
4 Circular motion and gravitation	2	4.1 (31)	–	–	4.1 (31)	–	–	–	4.1 (31)	4.2 (32)	4.2 (32)
5 Impulse and momentum	4	5.2 (36)	5.3 (37)	–	–	–	5.3 (37), 5.4 (38)	–	5.2 (36), 5.3 (37), 5.4 (38)	5.1 (34)	5.1 (34)
6 Center of mass	2	6.1 (39)	–	–	6.2 (40)	–	6.1 (39)	–	–	6.2 (40)	6.2 (40)
7 Work and energy	5	7.2 (43), 7.3 (46), 7.5 (49)	7.1 (42), 7.3 (46), 7.4 (48), 7.5 (49)	–	–	7.2 (43)	7.2 (43)	7.2 (43)	7.1 (42), 7.4 (48), 7.5 (49)	–	–
8a Rotational statics	2	8a.2 (52)	–	8a.2 (52)	8a.2 (52)	–	–	–	8a.1 (51)	–	–
8b Rotational dynamics and angular momentum	4	8b.1 (54), 8b.2 (55), 8b.3 (57), 8b.4 (59)	8b.3 (57), 8b.4 (59)	8b.1 (54)	–	8b.2 (55), 8b.4 (59)	8b.2 (55)	–	8b.1 (54), 8b.3 (57)	–	–

10 Simple harmonic motion	4	10.1 (62), 10.3 (64), 10.4 (66)	–	–	10.2 (63), 10.4 (66)	–	10.1 (62)	10.2 (63), 10.3 (64)	10.2 (63), 10.3 (64), 10.4 (66)	–	–
11a Fluid statics	2	11a.1 (68)	11a.2 (69)	–	11a.1 (68)	–	–	–	11a.1 (68), 11a.2 (69)	–	–
11b Fluid flow	2	11b.2 (71)	11b.1 (70)	–	–	–	11b.2 (71)	–	11b.1 (70)	–	11b.2 (71)
11c Fluids: statics and flow combined	2	11c.1 (73), 11c.2 (74)	–	–	11c.1 (73)	–	11c.2 (74)	–	–	–	–
Total	**45**	**26**	**17**	**2**	**9**	**6**	**10**	**7**	**24**	**5**	**6**

Formats. *Calculate or draw:* derive, calculate, or draw a diagram or graph. *Compare two cases:* say which of two situations has more of something, and why. *Rank:* order several related objects. *Predict a change:* say what happens to a quantity after something is changed. *Explain without equations:* reason in words first, then connect the words to a derivation. *Critique a student:* find what is right and wrong in a student's argument. *Does it make sense?:* test an equation or a graph against physics principles. *Paragraph response:* a clear, coherent, paragraph-length explanation. *Design an experiment:* write a procedure to measure a value or test a claim. *Analyze data:* graph data, often after choosing quantities that make a straight line, and use the slope or intercept.

1.1. (12 points, suggested time 25 minutes) A toy car is programmed to travel in the $+x$ direction in two phases of motion. In the first phase, the car travels with constant velocity v_1 for a time duration Δt_1. In the second phase, the toy car continues to travel, but now with constant nonzero acceleration. At the end of the second phase of motion, which lasts for a time duration Δt_2, the toy car reaches a final velocity v_2 that is greater than v_1. In a set of experimental trials, time durations Δt_1 and Δt_2 are varied while total time $\Delta t_{\text{TOT}} = \Delta t_1 + \Delta t_2$ and velocities v_1 and v_2 are held constant.

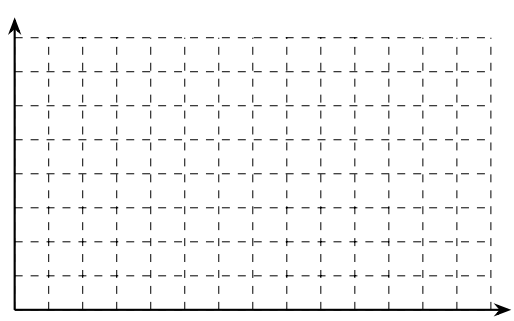
(a) i. Suppose the duration Δt_1 is much greater than the duration Δt_2. Estimate the x-displacement through which the car travels during the experiment. Express your answer in terms of v_1, v_2, Δt_{TOT}, and fundamental constants, as appropriate.

Briefly explain your reasoning without deriving or manipulating equations.

ii. Now suppose the duration Δt_1 is much less than the duration Δt_2. Estimate the x-displacement through which the car travels during the experiment. Express your answer in terms of v_1, v_2, Δt_{TOT}, and fundamental constants, as appropriate.

Briefly explain your reasoning without deriving or manipulating equations.

(b) Now suppose neither time duration Δt_1 nor time duration Δt_2 is much greater than the other, but that they are not necessarily equal. On the axes below, draw a velocity vs. time graph for the car. Label the axes and the values of times and velocities at the beginning and end of each phase of motion.



(c) Derive an equation for the x-displacement through which the car travels during the experiment in terms of v_1, v_2, Δt_1, Δt_{TOT}, and fundamental constants, as appropriate. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

(d) Consider the scenario from part (a)(ii), where the duration Δt_1 is much less than the duration Δt_2. Does your equation for the x-displacement of the car from part (c) agree with your reasoning in part (a)(ii)?

_____ Yes _____ No

Briefly explain your reasoning by addressing why, according to your equation, the x-displacement becomes (or approaches) a certain value when Δt_1 is much less than Δt_2.

(e) In one trial, the car travels through an overall displacement Δx, and the acceleration of the car during the second phase of motion is a_{I}. In a second trial, the car travels through the same overall displacement Δx through the same total time Δt_{TOT} and achieves the same final velocity v_2, but starts out with a slightly greater initial velocity v_1 (still less than v_2). The acceleration of the car during the second phase of motion during the second trial is a_{II}. How do the two accelerations a_{I} and a_{II} compare to each other?

_____ $a_{\mathrm{I}} > a_{\mathrm{II}}$ _____ $a_{\mathrm{I}} = a_{\mathrm{II}}$ _____ $a_{\mathrm{I}} < a_{\mathrm{II}}$

Briefly explain your reasoning. You may use a diagram or diagrams to support your explanation.

(f) For a trial in which the car travels through overall displacement Δx in total time Δt_{TOT}, starting with velocity v_1 and ending with velocity v_2, derive an expression for the acceleration a of the car during the second phase of motion. Express your answer in terms of Δx, v_1, v_2, Δt_{TOT}, and fundamental constants, as appropriate.

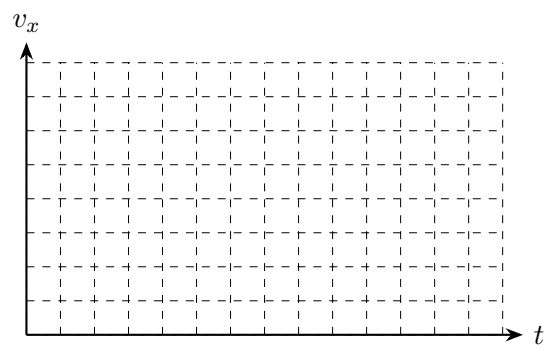
1.2. (7 points, suggested time 13 minutes) A car is moving in the $+x$ direction along a straight highway at speed v. At time $t = 0$, the driver applies the brakes so that the car slows down with constant acceleration until the car comes to rest. Let time $t_{1/2}$ denote the time at which the car's speed has decreased to half its original value, and let t_{STOP} denote the time at which the car is just then coming to rest.

(a) Is the car's x-displacement from time $t_{1/2}$ to time t_{STOP} greater than, less than, or the same as its x-displacement from time $t = 0$ to time $t_{1/2}$?

_____ Greater than _____ Less than _____ The same as

In a clear, coherent, paragraph-length response that does not derive or manipulate equations, explain your reasoning.

(b) On the axes below, sketch a graph of the car's velocity v_x as a function of time t from $t = 0$ until the car stops. Label the velocities v and $v/2$ on the vertical axis and the times $t_{1/2}$ and t_{STOP} on the horizontal axis.



(c) Derive the ratio of the car's x-displacement from time $t_{1/2}$ to time t_{STOP} to the car's x-displacement from time $t = 0$ to time $t_{1/2}$.

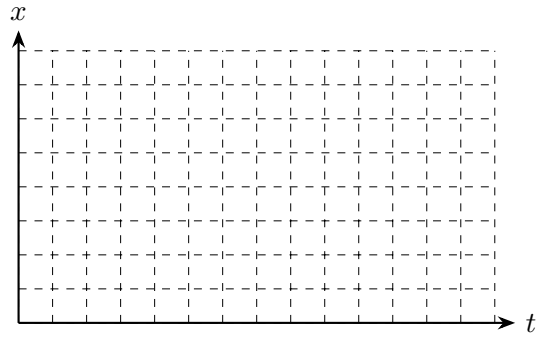
1.3. (7 points, suggested time 13 minutes) Let the $+x$ direction point along a straight racetrack from the start line ($x = 0$) to the finish line. At the start of a race, Car A and Car B are stopped at the start line. Each car almost immediately gets up to speed and then continues to move at constant velocity through the end of the race, which happens when Car A crosses the finish line first.

(a) Is Car A's average x-velocity during the race greater than, less than, or the same as Car B's average x-velocity during the race?

_____ Greater than _____ Less than _____ The same as

In a clear, coherent, paragraph-length response that incorporates physics principles, explain your reasoning.

(b) On the axes below, sketch graphs of position x as a function of time t for both cars during the race. Label each graph with the car's letter, and label the position x_F of the finish line and the time t_{END} at which the race ends.



2.1. (7 points, suggested time 13 minutes) A cannonball is launched in a slanted direction (partially upward and partially sideways) from the edge of a cliff above the ocean below. Air resistance is negligible.

(a) If the launch speed is doubled, with the launch direction unchanged, will the time of flight (from launch to splash) be less than doubled, exactly doubled, or more than doubled?

_____ Less than doubled _____ Exactly doubled _____ More than doubled

In a clear, coherent, paragraph-length response that incorporates physics principles, explain your reasoning.

(b) If the launch speed is doubled, with the launch direction unchanged, will the greatest height that the cannonball reaches above the launch point be less than doubled, exactly doubled, or more than doubled?

_____ Less than doubled _____ Exactly doubled _____ More than doubled

Briefly explain your reasoning.

2.2. (7 points, suggested time 13 minutes) A survey drone moves with constant velocity: its velocity has an upward vertical component of magnitude u and a horizontal component of magnitude w. When the drone is at height H above level ground, it releases a seed pod. Air resistance on the pod is negligible. A student writes the following equation for the horizontal distance D the pod travels between its release and the moment it reaches the ground:

$$D = w\sqrt{\frac{2H}{g}}$$

(a) Is the actual horizontal distance the pod travels greater than, less than, or equal to the value given by the student's equation?

_____ Greater than _____ Less than _____ Equal to

Briefly justify your answer.

(b) Derive an expression for the horizontal distance D the pod travels between its release and the moment it reaches the ground. Express your answer in terms of u, w, H, and physical constants, as appropriate.

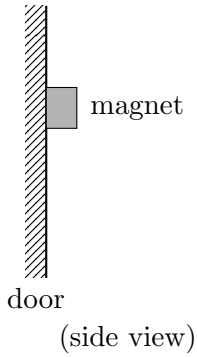
(c) Describe a condition on the drone's motion under which your expression from part (b) would agree with the student's equation. Use your expression to show that it agrees under that condition.

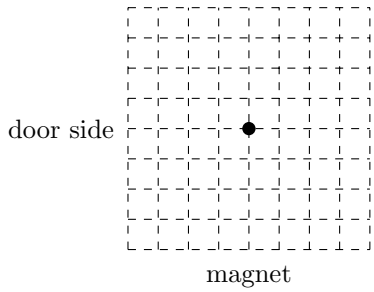
(d) On another survey, the drone releases an identical pod from the same height H while its velocity has the same horizontal component w but a *downward* vertical component of magnitude u. Is the horizontal distance this pod travels greater than, less than, or equal to the value given by the student's equation?

_____ Greater than _____ Less than _____ Equal to

Briefly justify your answer.

3.1. (7 points, suggested time 13 minutes) A small magnet of mass m clings to the flat vertical steel door of a refrigerator, as shown. The magnet does not move. The steel door exerts an attractive magnetic force on the magnet that is horizontal, points toward the door, and has magnitude F_M, which is greater than the magnet's weight. The coefficient of static friction between the magnet and the door is μ_S.





(a) On the dot above, which represents the magnet, draw and label the forces (not components) that act on the magnet. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Draw the relative lengths of the arrows to show which forces have equal magnitudes and which are larger.

(b) A student now presses the magnet straight toward the door with a finger, so that the finger exerts an additional horizontal force on the magnet. The magnet still does not move. Is the magnitude of the static friction force exerted by the door on the magnet greater than, less than, or the same as it was before the student pressed?

_____ Greater than _____ Less than _____ The same as

Briefly justify your answer.

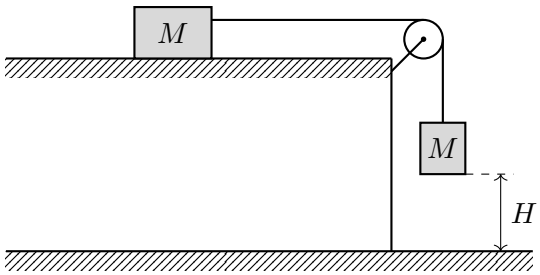
(c) While the student is still pressing, is the greatest possible magnitude of the static friction force that the door could exert on the magnet greater than, less than, or the same as it was before the student pressed?

_____ Greater than _____ Less than _____ The same as

Briefly justify your answer.

(d) The student removes the finger. A light hook is attached to the bottom of the magnet, and a calendar is hung from the hook so that the calendar hangs freely and does not touch the door. Derive an expression for the greatest mass m_c of calendar that can be hung without the magnet slipping. Express your answer in terms of m, F_M, μ_S, and physical constants, as appropriate.

3.2. (12 points, suggested time 25 minutes) In the figure below, a block of mass M on a frictionless, horizontal tabletop is connected to a taut massless string that runs over a frictionless, massless pulley and connects to a slotted mass set of mass M that is initially at height H above the floor. The block and slotted mass set are released from rest. The string stays taut, and all parts of the system consisting of the block, string, and slotted mass set have the same magnitude of acceleration a_1 until the slotted mass set hits the floor.



A second experiment is planned in which the slotted mass set will have mass $2M$. The block is unchanged. A student is asked to predict how the new magnitude of acceleration a_2 will compare with a_1. The student says:

> "Doubling the mass of the slotted mass set doubles the component of the net force on the system along the direction of acceleration. The block is the portion of the system trying to resist being accelerated, so keeping the same block means keeping the inertial mass of the system the same. According to Newton's second law, acceleration is proportional to net force and inversely proportional to inertial mass, so the magnitude of the acceleration will increase. Specifically, $a_2 = 2a_1$."

(a) i. Which aspects of the student's reasoning, if any, are correct? Explain how you arrived at your answer.

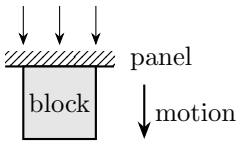
ii. Which aspects of the student's reasoning, if any, are incorrect? Explain how you arrived at your answer.

(b) Use quantitative reasoning, including equations as needed, to develop an expression for a_2. Express your answer in terms of a_1.

(c) i. Explain how any correct aspects of the student's reasoning that you identified in part (a) are expressed by your mathematical relationships in part (b). Refer to the relationships you wrote in part (b), not only to your final answer.

ii. Explain how your relationships in part (b) correct any incorrect aspects of the student's reasoning that you identified in part (a). Refer to the relationships you wrote in part (b), not only to your final answer.

3.3. (7 points, suggested time 13 minutes) In the experiment shown below, a block near Earth's surface is pressed from above by a movable panel. The block is gaining speed while moving downward with a constant acceleration. Air resistance is negligible. The experiment is repeated, but now with the normal force exerted by the panel on the block only half as strong as before.



(a) Will the magnitude of the resulting acceleration of the block be greater than, less than, or equal to half the original magnitude of acceleration of the block?

_____ Greater than half _____ Less than half _____ Equal to half

In a clear, coherent, paragraph-length response that incorporates physics principles, explain your reasoning.

(b) The experiment is repeated a third time, now with the normal force exerted by the panel on the block twice as strong as in the original experiment. Will the magnitude of the acceleration of the block be greater than, less than, or equal to twice the original magnitude of acceleration of the block?

_____ Greater than twice _____ Less than twice _____ Equal to twice

Briefly explain your reasoning.

3.4. (7 points, suggested time 13 minutes) The equivalence principle states that, for any given object, the inertial mass and the gravitational mass have equal values. Students are asked to test the equivalence principle for a cart, using the resources listed in the table below.

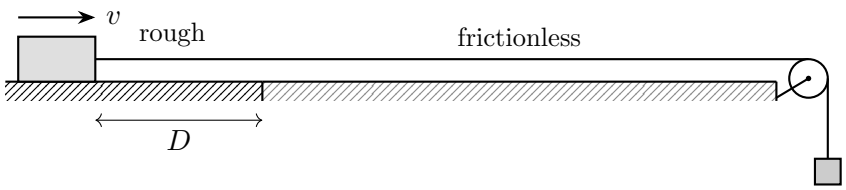
Resource	**Description**
Cart	Has massless wheels that spin without friction
Floor	Smooth, firm, horizontal, flat surface
Motion detector	Mounted horizontally; tracks the horizontal position of an object on the floor and produces position–time, velocity–time, and acceleration–time graphs on a computer
Force probe	Displays the strength of the pulling force exerted by the probe on whatever object is attached to its hook
Classmates	Other students can hold and read items for you

(a) Describe an experimental procedure that could provide data that could be used to test whether the equivalence principle is satisfied by the cart. Give enough detail so that another student could carry out your procedure. Be sure to provide a way to reduce experimental uncertainty.

(b) Describe an analytic procedure that could be applied to the data obtained using the method you described in part (a) to determine whether the equivalence principle is satisfied by the cart. Give enough detail so that another student could perform your analysis.

(c) Using physics principles, explain why the analytic procedure you described in part (b) would work.

3.5. (7 points, suggested time 13 minutes) In the figure below, a block of mass m is connected to a massless string draped over a massless, frictionless pulley. The other end of the string is connected to a slotted mass set, also of mass m. The block is on the rough portion (darker shading) of a horizontal tabletop, and the coefficients of kinetic and static friction between the block and the rough portion of the table are $\mu_K = \mu_S = 2.0$. The block is initially skidding to the right with speed $v = 10$ m/s at a distance $D = 7.5$ m from the location where the rough portion of the tabletop joins the frictionless portion (lighter shading).



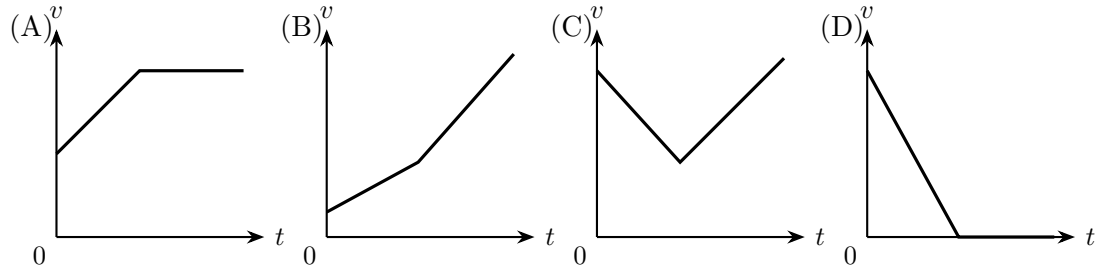
Treat the block as small compared with D. The tabletop is long, and the block does not reach the pulley during the motion described below.

(a) Calculate the magnitude of the acceleration of the block while it is on the rough portion of the tabletop.

(b) Calculate the speed of the block when it reaches the frictionless portion of the tabletop, or, if it stops before reaching it, calculate how far short of the frictionless portion it stops.

(c) Calculate the magnitude of the acceleration the block would have while moving to the right on the frictionless portion of the tabletop, and state whether the block would be speeding up or slowing down there.

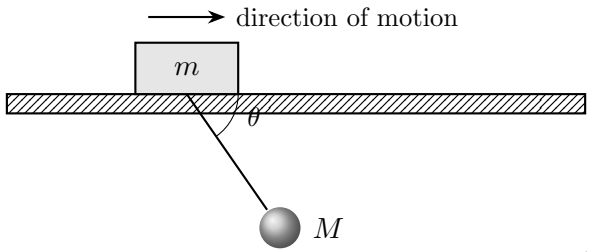
(d) A video recording is analyzed to obtain a speed–time graph for the block, starting at the instant shown in the figure and ending before the block reaches the pulley. Which one of the following graphs represents the speed–time graph for the block?



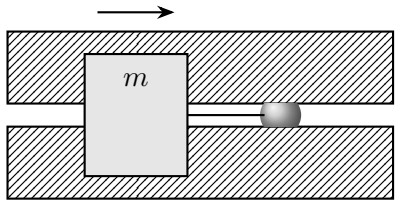
_____ A _____ B _____ C _____ D

Briefly explain how you arrived at your choice.

3.6. (12 points, suggested time 25 minutes) As shown below, a block of mass m rests across two rough horizontal planks that are separated by a narrow gap (see the side and top views). The block is attached to a massless string that passes down through the gap; the other end of the string is connected to a sphere of mass M that hangs below the planks. The coefficient of kinetic friction between the block and the planks is μ_K. It is possible (perhaps after some practice trials) to launch the block, string, and sphere to the right so that the angle θ between the taut string and the horizontal has a constant value during the time interval when the block, string, and sphere are still moving together and have not yet come to rest. The acceleration of the block–string–sphere system before coming to rest has magnitude a. The sphere does not touch the planks, and air resistance is negligible.



side view



top view (string and sphere seen through the gap)

(a) i. While the block, string, and sphere move together, the planks exert a normal force of magnitude $(m + M)g$ on the block, even though the block's mass is only m. Explain why.

Suppose that the experiment is repeated, but now with mass M doubled.

ii. Does the magnitude a of the acceleration of the block–string–sphere system increase, decrease, or stay the same?

_____ Increase _____ Decrease _____ Stay the same

Briefly explain your reasoning without manipulating equations. (You may write, cite, and describe features of equations, but do not merely write, "I solved the equations, and the final expression shows that . . . ")

iii. Does the angle θ increase, decrease, or stay the same?

_____ Increase _____ Decrease _____ Stay the same

Briefly explain your reasoning without manipulating equations.

(b) i. Derive an expression for the magnitude a of the acceleration in terms of m, M, μ_K, and physical constants, as appropriate.

ii. Derive an expression for the angle θ in terms of m, M, μ_K, and physical constants, as appropriate.

A class of students performed this experiment. There was only one set of equipment, so groups took turns. The table shows the angle θ measured by each group using video analysis.

Group	1	2	3	4	5	6	7
Time group finished	10:02	10:07	10:12	10:18	10:22	10:30	10:37
Angle θ ($\pm 2°$)	52	51	54	55	58	63	61

(c) i. Use your result from part (b)(ii) to calculate the coefficient of kinetic friction μ_K for Group 1 and for Group 7.

ii. Taking into account the stated uncertainty in the angle measurements, do the data show that μ_K was different for Group 7 than for Group 1?

_____ Yes _____ No

Briefly justify your answer.

iii. The students notice that the angle θ mostly increases from group to group. One student accuses the class of sabotage: each group shaved some mass off the block, so the block pressed on the planks less and less, which reduced the kinetic friction force on it. The student argues that weaker friction on the block gave the system a smaller acceleration and, thus, a more vertical string. Is the student's accusation consistent with your work in part (b)?

_____ Yes _____ No

Briefly explain.

iv. Propose an alternative explanation for the trend of increasing angle θ, and use your results from part (b) and part (c)(i) to support it.

3.7. (12 points, suggested time 25 minutes) A student claims that the maximum static friction force $f_{S,\max}$ that two surfaces can exert before they slip increases in proportion to the normal force between the surfaces until it reaches 8.0 N, and that $f_{S,\max}$ stays equal to 8.0 N for all greater normal forces.

(a) Describe an experimental setup that could be used to test the student's claim for a wooden block and a wooden board. You may use any equipment ordinarily available in a typical physics classroom laboratory. Include a diagram of your setup.

(b) Describe how you would use your setup to obtain data to test the student's claim. Indicate which equipment is used to measure each relevant quantity. Give enough detail so that another student could carry out your procedure, and include a way to reduce uncertainty.

(c) Describe how you could analyze your data to test the student's claim. Include a way to take measurement uncertainty into account.

(d) Using physics principles, explain how you know that your analysis method could be used to test the student's claim.

The student performs an experiment using a wooden block and a wooden board and obtains the data in the table below.

Normal force (± 0.02 N)	Maximum static friction force (± 0.1 N)
2.00	1.6
4.00	3.0
8.00	5.9
12.00	8.9
16.00	12.0
20.00	15.1

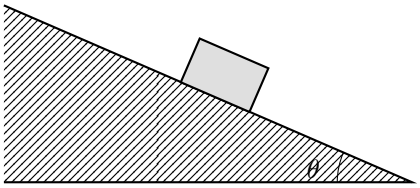
(e) Are the data in the table consistent with the student's claim?

_____ Yes _____ No

Briefly justify your answer.

(f) Use the data in the table to determine the coefficient of static friction between the wooden block and the wooden board.

3.8. (7 points, suggested time 13 minutes) In the figure below, a block is sliding on a fixed inclined plane with angle of inclination θ. The block might be sliding up the plane, or it might be sliding down the plane.



(a) Suppose the plane is frictionless. Is the magnitude of the block's acceleration greater while the block slides up the plane, greater while it slides down the plane, or the same in both cases?

_____ Greater sliding up _____ Greater sliding down _____ The same

Briefly explain your reasoning.

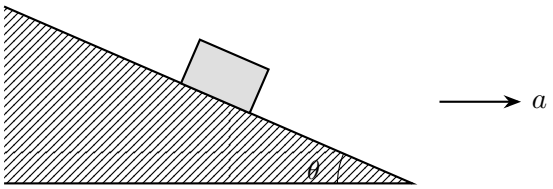
(b) Now suppose there is friction between the plane and the block, and the friction force has the same magnitude whether the block slides up or down. Is the magnitude of the block's acceleration greater while the block slides up the plane, greater while it slides down the plane, or the same in both cases?

_____ Greater sliding up _____ Greater sliding down _____ The same

In a clear, coherent, paragraph-length response that incorporates physics principles, explain your reasoning.

(c) The coefficient of kinetic friction between the block and the plane is μ_K. The block is launched up the plane with speed v_0. Derive an expression for the distance the block slides up the plane before it momentarily stops. Express your answer in terms of v_0, θ, μ_K, and physical constants, as appropriate.

3.9. (7 points, suggested time 13 minutes) In the figure below, the frictionless inclined plane has angle of inclination θ and is accelerating purely horizontally to the right, with acceleration of magnitude a. As a result, the box remains at a constant height on the plane, rather than sliding up or down along it. In a second experiment, the magnitude a of the acceleration is doubled.



(a) For the box again to remain at a constant height on the plane, will the angle of inclination need to be more than doubled, less than doubled, or exactly doubled?

_____ More than doubled _____ Less than doubled _____ Exactly doubled

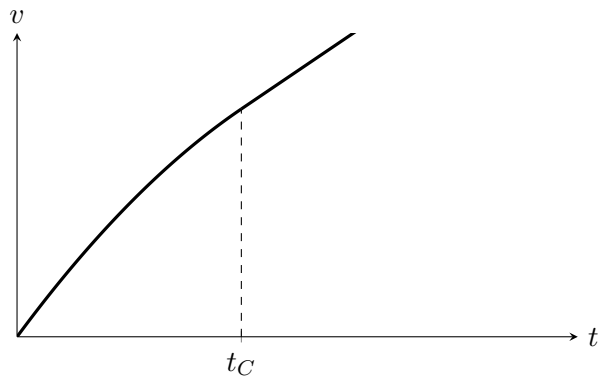
In a clear, coherent, paragraph-length response that incorporates physics principles, explain your reasoning.

(b) With the angle adjusted so that the box remains at a constant height, is the magnitude of the normal force exerted by the plane on the box in the second experiment greater than, less than, or equal to its magnitude in the first experiment?

_____ Greater than _____ Less than _____ Equal to

Briefly explain your reasoning.

3.10. (7 points, suggested time 13 minutes) At time $t = 0$, a massive object is released from rest and falls toward the Earth through a uniform fluid in which drag is not negligible. The buoyant force on the object is negligible. The graph shows the object's speed v as a function of time t. The graph consists of an initial portion, from $t = 0$ to $t = t_C$, that is curved (concave down), and a second portion, from $t = t_C$ onward, that is a straight line.



(a) A student claims that the magnitude F_{DRAG} of the drag force exerted by the fluid on the object increases whenever the object's speed v through the fluid increases. Does the graph support the student's claim?

_____ Yes _____ No

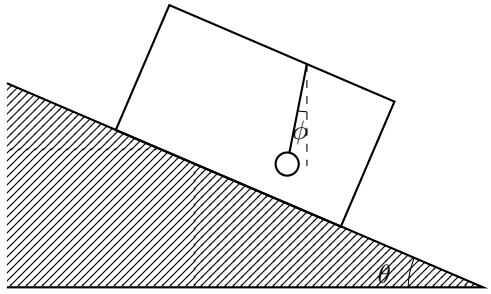
In a clear, coherent, paragraph-length response that incorporates physics principles, explain your answer.

(b) Is the magnitude of the drag force on the object at time $t = 2t_C$ greater than, less than, or equal to its magnitude at time $t = \frac{1}{2}t_C$?

_____ Greater than _____ Less than _____ Equal to

Briefly explain your reasoning.

3.11. (7 points, suggested time 13 minutes) A boxcar slides along a ramp with angle of elevation θ. A massless string connects a massive ball to the ceiling of the boxcar. While the boxcar slides, the ball hangs at rest relative to the boxcar, with the string making a constant angle ϕ with the vertical. (The figure is not to scale; the angle drawn is not necessarily the actual angle.)



(a) Suppose the ramp is frictionless. How does ϕ compare with θ?

_____ $\phi > \theta$ _____ $\phi = \theta$ _____ $\phi < \theta$ _____ Cannot be determined

Briefly explain your reasoning.

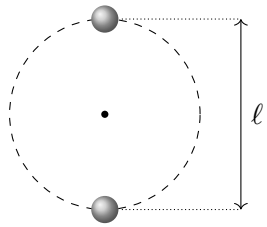
(b) Now there is friction between the ramp and the boxcar. The boxcar is sliding *up* the ramp, and the string makes a constant angle ϕ' with the vertical. How does ϕ' compare with θ?

_____ $\phi' > \theta$ _____ $\phi' = \theta$ _____ $\phi' < \theta$ _____ Cannot be determined

In a clear, coherent, paragraph-length response that incorporates physics principles, explain your reasoning.

(c) The coefficient of kinetic friction between the ramp and the boxcar is μ_K. While the boxcar slides up the ramp as in part (b), with the ball hanging at rest relative to the boxcar, derive an expression for the magnitude of the boxcar's acceleration. Express your answer in terms of θ, μ_K, and physical constants, as appropriate.

4.1. (7 points, suggested time 13 minutes) A pair of stars, each having mass m, orbit a point midway between them. The distance between the stars is ℓ.



(a) If the mass of each star is increased by the same factor, with the distance ℓ unchanged, how will the orbital speed of each star be affected?

_____ Increase _____ Decrease _____ Remain the same

In a clear, coherent, paragraph-length response that may also contain equations, justify your answer.

(b) Derive an expression for the orbital speed v of each star in terms of m, ℓ, and fundamental constants, as needed.

4.2. (12 points, suggested time 25 minutes) An astronomer discovers a new planet that is circularly orbited by four moons. The astronomer obtains the orbital radii and periods for the four moons in the table below.

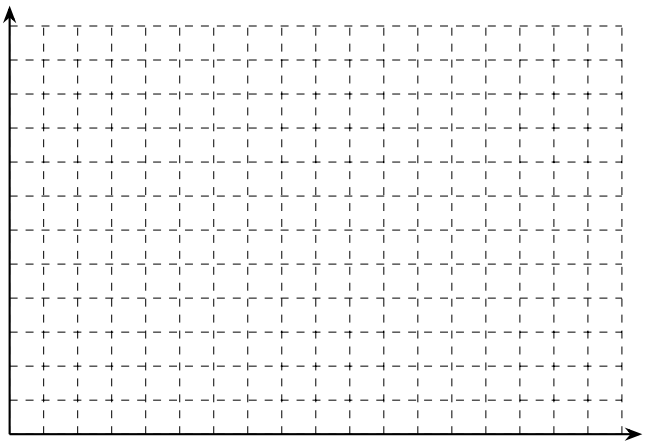
Moon	Orbital radius R (m) $\pm 3 \times 10^7$	Orbital period T (s) $\pm 2 \times 10^3$		
A	6.2×10^8	2.66×10^5		
B	9.2×10^8	4.80×10^5		
C	1.07×10^9	4.26×10^5		
D	1.20×10^9	5.06×10^5		

One of the astronomer's colleagues claims that practically all the mass in the vicinity of the newly discovered planet and its four moons is concentrated in a point-like fashion in the center of the planet itself.

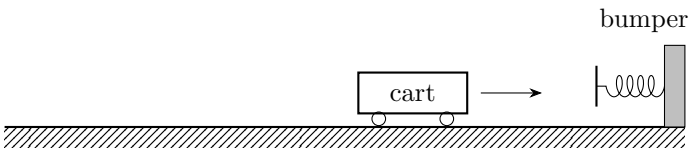
(a) Provide a method for analyzing the data to test the colleague's claim. Be sure to provide enough detail so that another physics student could carry out your analysis procedure. Provide a method to take experimental uncertainty into account.

(b) Use physics principles to explain why the analysis method you described in part (a) can be used to test the colleague's claim.

(c) Carry out the method you described in part (a) and report whether the colleague's claim is correct. You may use the grid below and the empty columns in the data table if needed for your analysis.



5.1. (12 points, suggested time 25 minutes) A student claims: "The impulse that a spring bumper delivers to a cart while the cart bounces off the bumper depends only on how fast the cart is moving just before it reaches the bumper. It does not depend on the mass of the cart."
The student has a cart of mass 0.25 kg, several bars that can be fastened to the cart (each bar has mass 0.25 kg), and a level, low-friction track with a spring bumper fixed at one end, as shown. Other equipment that is usually found in a school physics laboratory is also available, but there is no device that measures force.



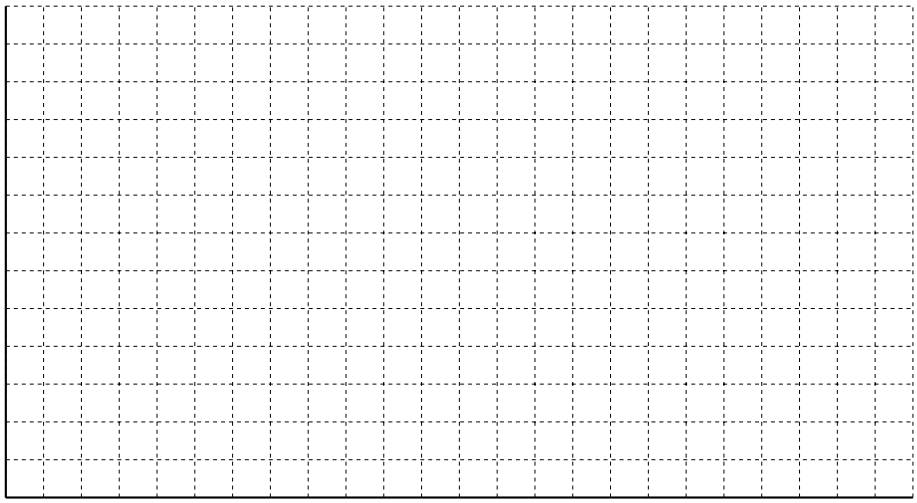
(a) Describe an experimental procedure that the student could use to collect data to test the claim. Include a labeled diagram if it helps. State which equipment is used to measure each quantity, and give enough detail that another student could carry out your procedure.

(b) Describe how the data from your procedure in part (a) could be analyzed, including a graph if you would use one, to decide whether the data support the claim.

The student carries out an experiment. In every trial, the cart approaches the bumper with speed 0.80 ± 0.02 m/s. The student finds the impulse delivered by the bumper from the cart's measured velocities just before and just after the bounce. The results are shown in the table.

Trial	Mass of cart and bars (kg)	Magnitude of impulse (N·s)
1	0.25	0.36
2	0.50	0.68
3	0.75	1.07
4	1.00	1.38
5	1.25	1.77

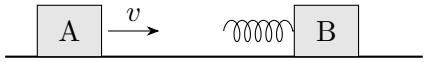
(c) On the grid below, plot the data and draw a straight line that best fits the data. Label the axes with quantities and units, and choose scales.



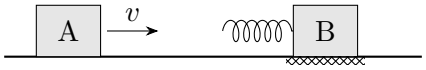
(d) Do the data support the student's claim? Justify your answer by referring to your graph.

(e) Assume that in every trial the cart leaves the bumper with the same speed. Use your best-fit line to determine that speed.

5.2. (7 points, suggested time 13 minutes) Blocks A and B have equal mass m and slide on a frictionless, horizontal surface. A light spring of spring constant k is attached to the left side of block B. Block A slides toward block B with speed v, compresses the spring, and later moves away. The experiment is done twice. In Experiment 1, block B is initially at rest and free to slide. In Experiment 2, block B is glued to the surface so that it cannot move. Block A has the same initial speed v in both experiments.



Experiment 1: B free to slide



Experiment 2: B glued in place

(a) Is the greatest compression of the spring in Experiment 1 greater than, less than, or the same as the greatest compression of the spring in Experiment 2?

_____ Greater than _____ Less than _____ The same as

In a clear, coherent paragraph-length response that may also contain diagrams and/or equations, justify your answer.

(b) Calculate the ratio $\frac{x_1}{x_2}$ of the greatest compression x_1 in Experiment 1 to the greatest compression x_2 in Experiment 2.

5.3. (7 points, suggested time 13 minutes) On a horizontal, frictionless track, cart A is launched toward cart B, which is identical to cart A and initially at rest. The two carts stick together immediately upon contact. Charlie tries to use conservation of momentum to determine the final speed of the stuck-together carts, but Charlie misremembers the formula for x-momentum as $p_x = \frac{1}{2}mv_x^2$.

(a) Is the speed of the stuck-together carts that Charlie calculates greater than, less than, or equal to the speed Charlie should have calculated?

_____ Greater than _____ Less than _____ Equal to

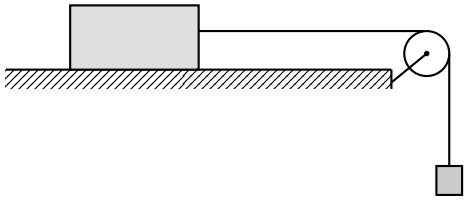
In a clear, coherent, paragraph-length response that incorporates physics principles, explain your reasoning.

(b) Is the total kinetic energy of the two carts just after the collision greater than, less than, or equal to their total kinetic energy just before the collision?

_____ Greater than _____ Less than _____ Equal to

Briefly explain your reasoning.

5.4. (7 points, suggested time 13 minutes) A group of students is shown a block that is connected to a taut massless string. The string passes over a massless, frictionless pulley and is connected to a slotted mass set, as shown in the figure below. The block is on a horizontal, frictionless tabletop.

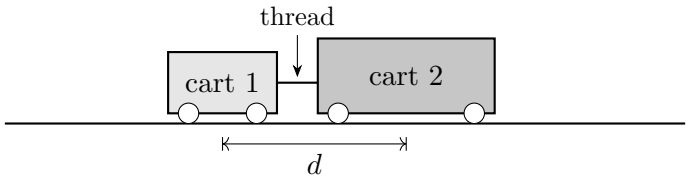


Initially, an instructor holds the block in place. The students are asked to predict how the horizontal momentum of the system consisting of the block, string, and slotted mass set will (or will not) change after the instructor releases the block. A group of students complains that there is an apparent contradiction:

> *Without the gravitational force that the Earth exerts on the slotted mass set, the system would remain at rest, so that gravitational force is the cause of the motion of the system. The gravitational force on the slotted mass set is directed straight down, so the horizontal component of the impulse on the system is zero. Thus, the total horizontal momentum of the system should stay constant. However, after the block is released, the block moves faster and faster toward the right (while the slotted mass set moves faster and faster downward), so the total horizontal momentum of the system actually increases in magnitude.*

In a clear, coherent, paragraph-length response that refers to the horizontal component of the impulse on the system, explain why the total horizontal momentum of the system changes after the block is released, even though the gravitational force exerted by the Earth on the slotted mass set is purely vertical.

6.1. (7 points, suggested time 13 minutes) Cart 1, of mass m, and cart 2, of mass $3m$, sit on a long horizontal track with negligible friction. Each cart carries a magnet, and the magnets repel each other. A thread ties the carts together so that they are at rest with their centers a distance d apart. The thread is then burned, and the carts move apart.



(a) Derive an expression for the distance from the center of cart 1 to the center of mass of the two-cart system before the thread is burned. Express your answer in terms of d.

A student makes the following claim about what happens after the thread is burned:

> "The magnets push on each cart with forces of equal size, so the lighter cart 1 speeds away faster than cart 2. Because cart 1 moves much farther than cart 2, the center of mass of the two-cart system moves in the direction that cart 1 moves."

(b) i. Identify one aspect of the student's claim that is correct, and briefly explain why it is correct.

ii. Identify one aspect of the student's claim that is incorrect, and explain why it is incorrect.

(c) At one instant after the thread is burned, cart 1 is a distance s from where it started. Derive an expression for the distance cart 2 is from where it started at that instant. Express your answer in terms of s.

6.2. (12 points, suggested time 25 minutes) Two pucks float on an air-hockey table, so friction is negligible. Each puck carries a ring magnet, and the magnets repel, so the pucks push each other apart without touching. Puck 1 has mass $m_1 = 0.20$ kg, and puck 2 has mass $m_2 = 0.30$ kg. Puck 1 is launched straight toward puck 2, which is initially at rest. A student claims:

> "The center of mass of the two-puck system moves with constant velocity before, during, and after the pucks interact."

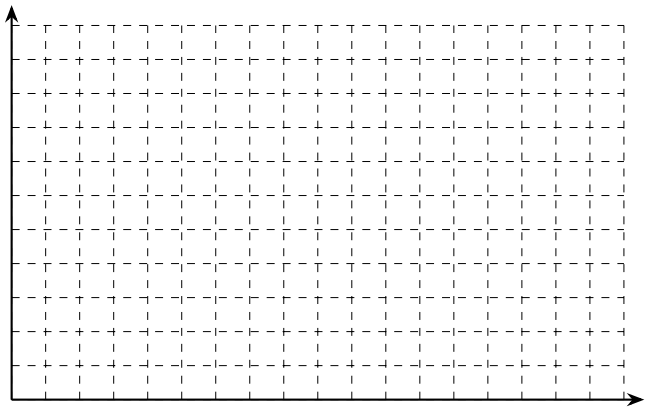
(a) The students have a video camera mounted above the table, a meter stick, and a balance. Describe an experimental procedure that could be used to collect data to test the claim. Give enough detail that another student could carry out your procedure.

A group carries out a procedure and obtains the positions x_1 and x_2 of the centers of the pucks along a line on the table, measured to within ± 0.002 m. The magnets begin to push on each other when the centers are about 0.10 m apart; the pucks are small enough that they never touch.

t (s)	x_1 (m)	x_2 (m)	
0.0	0.101	0.449	
0.2	0.199	0.450	
0.4	0.300	0.451	
0.6	0.399	0.451	
0.8	0.445	0.486	
1.0	0.433	0.562	
1.2	0.414	0.641	
1.4	0.392	0.722	
1.6	0.374	0.800	

(b) i. Use the empty column in the table to record a quantity that can be graphed to test the claim. Label the column.

ii. On the grid below, plot that quantity as a function of time. Clearly scale and label all axes.



iii. Is the claim supported by the data?

_____ Yes _____ No

Briefly explain how your graph supports your answer.

(c) Use your graph to determine the velocity of the center of mass. Compare it with the value predicted from the masses and puck 1's velocity before the interaction.

(d) Another group analyzes the same data but mistakenly swaps the two masses, using 0.30 kg for puck 1 and 0.20 kg for puck 2 in the quantity they graph.

i. Using the data, calculate the slope of that group's graph from $t = 0.0$ s to $t = 0.6$ s and from $t = 1.0$ s to $t = 1.6$ s.

ii. Would that group's graph be a single straight line?

_____ Yes _____ No

Briefly explain your reasoning, including why it matters which mass is used for which puck.

7.1. (7 points, suggested time 13 minutes) A block of mass m is initially at rest on a horizontal, frictionless track. The block is pushed toward the right by a force of constant strength F through a distance $\Delta\ell$, immediately after which the speed of the block is v_1. In a second experiment, a block of mass $3m$ is initially at rest on the same track and then pushed toward the right by a force, again of constant strength F, again through a distance $\Delta\ell$, immediately after which the speed of the block of mass $3m$ is v_2. Indicate how the speeds v_1 and v_2 compare.

_____ $v_1 > v_2$ _____ $v_1 < v_2$ _____ $v_1 = v_2$

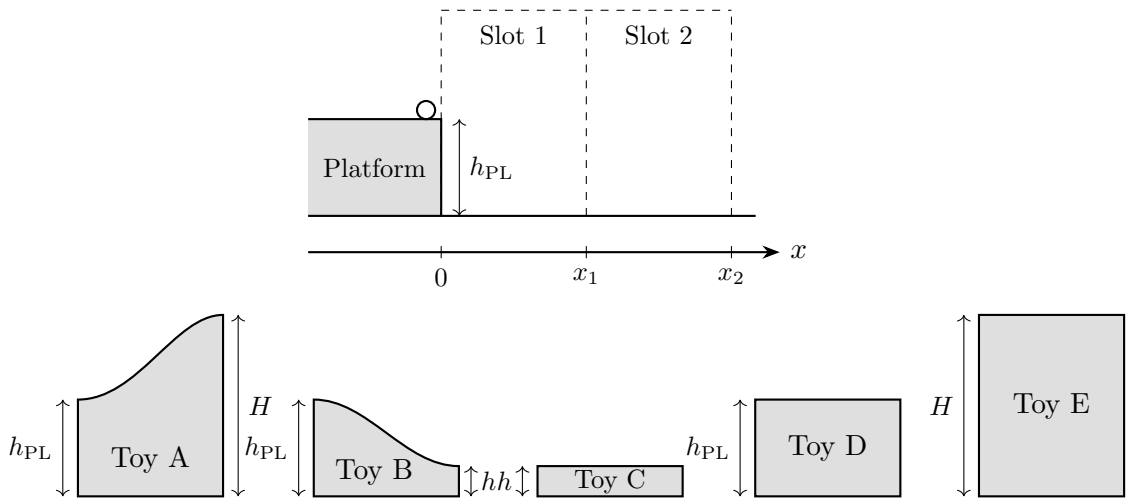
In a clear, coherent, paragraph-length response, explain your choice in terms of energy principles.

7.2. (12 points, suggested time 25 minutes) A small frictionless ball slides with speed v_0 across a horizontal platform at height h_{PL} above the ground toward two regions of equal horizontal width labeled Slot 1 and Slot 2. A group of children is offered one copy each of toys A–E, which they can insert into the slots. The children are not allowed to rotate the toys. The children are instructed to insert one toy into each slot so as to satisfy two conditions.

Condition 1: The speed of the ball just as the ball passes horizontal position $x = x_2$ must have the smallest possible value.

Condition 2: From among those toy selections that satisfy condition 1, the children must then choose the assignment of toys to slots that minimizes the time the ball takes to travel from horizontal position $x = 0$ to horizontal position $x = x_2$.

At all times until passing horizontal position $x = x_2$, the ball must be in contact with the platform or a toy. The top surfaces of Toys A and B are level at their left and right edges. Assume that v_0 is great enough for the ball to reach $x = x_2$, and that the ball stays in contact with any toy surface it slides along without a step.



All toys have the same width as a slot. Heights: $h < h_{\mathrm{PL}} < H$.

(a) i. Using energy principles, explain why satisfying condition 1 requires the ball to be as high as possible as it passes $x = x_2$.

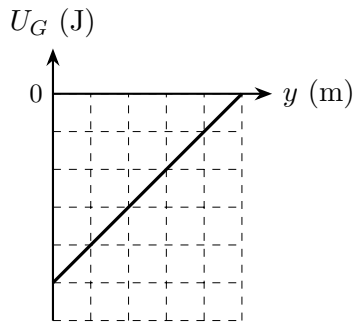
ii. Explain why every assignment of toys to slots that satisfies condition 1 must include Toy A.

iii. Now considering also condition 2, which toys should the children place in the slots?

Slot 1 should contain toy _____ Slot 2 should contain toy _____

Briefly explain your reasoning.

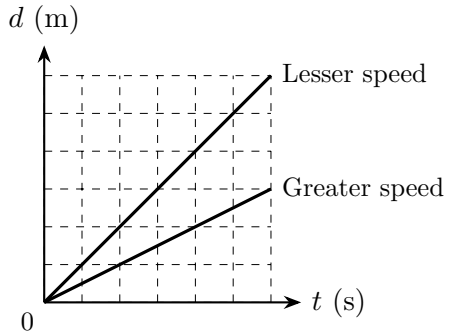
(b) On the Internet, the children find the following graph for the gravitational potential energy of an object–Earth system as a function of object height y above the Earth. Regardless of whether the graph is actually correct, is the graph consistent with your reasoning in part (a)?



_____ Consistent _____ Inconsistent

Briefly explain your reasoning.

(c) On the Internet, the children also find the following graph relating the distance traveled to the elapsed travel time and the speed at which travel proceeds. Regardless of whether the graph is actually correct, is the graph consistent with your reasoning in part (a)?



_____ Consistent _____ Inconsistent

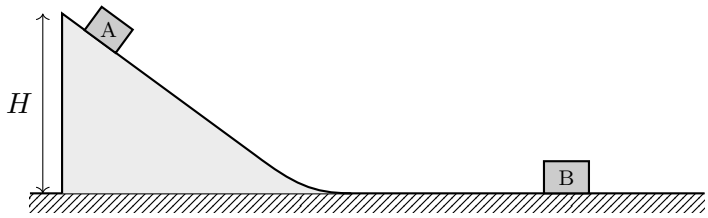
Briefly explain your reasoning.

(d) One of the children attempts to compute the x-velocity $v_{x,1}$ that the ball would have at $x = x_1$ if Toy B were placed in Slot 1. The child argues that the speed at $x = x_1$ can be computed using the equation $v_{x,f}^2 = v_{x,i}^2 + 2a_x\Delta x$ by letting the initial x-velocity be v_0, the final x-velocity be $v_{x,1}$, recognizing that the acceleration is $-g$, and recognizing that the displacement is $\Delta x = h - h_{\mathrm{PL}}$. Identify a feature of the child's reasoning, if any, that is incorrect. If the child's reasoning is correct, say so. Justify your answer.

(e) Consider the assignments of toys that satisfy condition 1. Derive an expression for the difference between the times the ball takes to travel from $x = 0$ to $x = x_2$ for these assignments. Express your answer in terms of v_0, x_1, H, h_{PL}, and physical constants, as appropriate.

(f) Suppose Toy B is placed in Slot 1. Derive an expression for the speed of the ball as it passes horizontal position $x = x_1$. Express your answer in terms of v_0, h_{PL}, h, and physical constants, as appropriate.

7.3. (7 points, suggested time 13 minutes) Block A, of mass m_A, is released from rest at a height H on a frictionless inclined ramp that is attached to a horizontal, frictionless floor. Block B, of mass m_B, initially rests on the floor. Take the gravitational potential energy of each block–Earth system to be zero when that block is on the floor.



(a) At the moment block A is released, how does the gravitational potential energy $U_{G,\text{A\&E}}$ of the block A–Earth system compare with the gravitational potential energy $U_{G,\text{B\&E}}$ of the block B–Earth system?

_____ $U_{G,\text{A\&E}} > U_{G,\text{B\&E}}$ _____ $U_{G,\text{A\&E}} < U_{G,\text{B\&E}}$ _____ $U_{G,\text{A\&E}} = U_{G,\text{B\&E}}$

Briefly explain your reasoning.

The foot of the ramp is smoothly curved, so block A moves smoothly onto the floor. Block A then collides with and sticks to block B.

(b) Derive an expression for the kinetic energy of the stuck-together blocks just after the collision. Express your answer in terms of m_A, m_B, H, and physical constants, as appropriate.

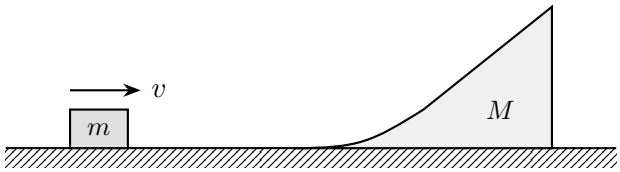
(c) The experiment is repeated, but now block A has a greater mass m_A, and everything else is unchanged. Is the kinetic energy K_2 of the stuck-together blocks in the new experiment greater than, less than, or equal to the kinetic energy K_1 of the stuck-together blocks in the original experiment?

_____ $K_2 > K_1$ _____ $K_2 < K_1$ _____ $K_2 = K_1$

Briefly explain your reasoning.

(d) Derive an expression for the amount of mechanical energy converted to internal energy in the collision. Express your answer in terms of m_A, m_B, H, and physical constants, as appropriate.

7.4. (7 points, suggested time 13 minutes) As shown in the figure below, a block of mass m slides with speed v across a horizontal, frictionless surface toward a wedge of mass $M > m$ that is initially at rest on the surface and free to slide on it. The wedge's inclined top surface is frictionless and smoothly joins the horizontal surface. The block slides up the wedge, without reaching the top, and reaches a maximum height H. The experiment is repeated with the same initial speed v, but now the block has mass M and the wedge has mass m.



(a) Is the speed of the wedge at the instant the block reaches its maximum height in the second experiment greater than, less than, or equal to the speed of the wedge at that instant in the first experiment?

_____ Greater than _____ Less than _____ Equal to

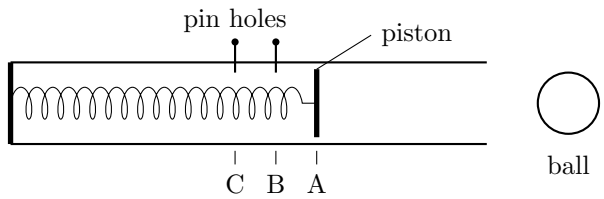
Briefly explain your reasoning.

(b) Will the new maximum height of the block on the wedge be greater than, less than, or equal to H?

_____ Greater than _____ Less than _____ Equal to

In a clear, coherent, paragraph-length response that incorporates physics principles, explain your reasoning.

7.5. (7 points, suggested time 13 minutes) A horizontally oriented launcher (below) consists of a massless spring and an attached thin, massless piston housed in a frictionless tube that is capped at one end. When the piston is at position A, the spring is relaxed. The spring can be compressed by using thin pins to hold the piston at positions B and C.



(a) In one experiment, a ball is pushed into the launcher, and a pin keeps the spring compressed with the ball at rest touching the piston, with the piston at position B. In a second experiment, the same procedure is used, except that the piston is held at position C. While the ball is held at rest in this way, how does the mechanical energy $E_{\text{MECH,B}}$ of the ball in the first experiment compare with the mechanical energy $E_{\text{MECH,C}}$ of the ball in the second experiment?

_____ $E_{\text{MECH,C}} > E_{\text{MECH,B}}$ _____ $E_{\text{MECH,C}} < E_{\text{MECH,B}}$ _____ $E_{\text{MECH,C}} = E_{\text{MECH,B}}$

Briefly explain your reasoning.

The launcher is now used to launch the ball. First, the launcher is horizontal, and the ball is launched from rest with the piston released from position B. The ball leaves the piston with speed v_{HOR} as the piston passes position A. In a second trial, the ball is again launched from rest with the piston released from position B, but now with the launcher pointing vertically upward. The ball leaves the piston with speed v_{VER} as the piston passes position A.

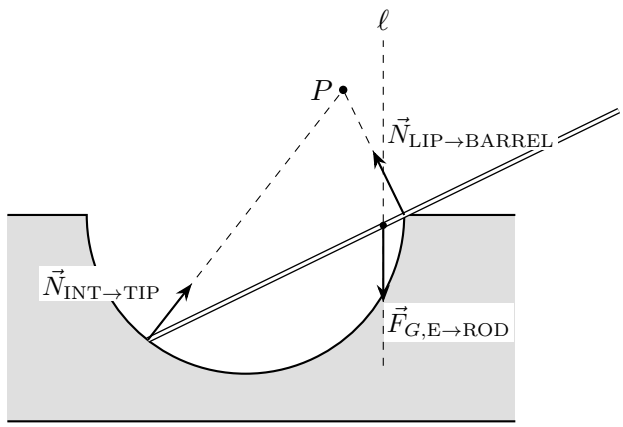
(b) How do the launch speeds v_{HOR} and v_{VER} compare?

_____ $v_{\text{HOR}} > v_{\text{VER}}$ _____ $v_{\text{HOR}} < v_{\text{VER}}$ _____ $v_{\text{HOR}} = v_{\text{VER}}$

In a clear, coherent, paragraph-length response that incorporates physics principles, explain your reasoning.

(c) The spring constant is 200 N/m, the distance between positions A and B is 0.050 m, and the ball's mass is 0.050 kg. Calculate v_{VER}.

8a.1. (7 points, suggested time 13 minutes) The figure below shows a cutaway view of a frictionless hemispherical bowl and a rod. The tips of the rod and the lip of the bowl are rounded and *slightly* elastic. As a result, the normal force $\vec{N}_{\text{INT}\to\text{TIP}}$ that the interior of the bowl exerts on a tip of the rod is perpendicular to the interior of the bowl, and the normal force $\vec{N}_{\text{LIP}\to\text{BARREL}}$ that the lip of the bowl exerts on the barrel of the rod is perpendicular to the rod. Lines drawn through these two normal force vectors intersect at point P. Vertical line ℓ is drawn through the gravitational force vector $\vec{F}_{G,\text{E}\to\text{ROD}}$ acting at the center of mass of the rod. The forces are not drawn to scale, and the figure might not show the rod in static equilibrium.

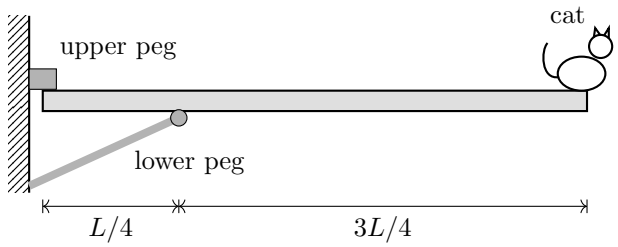


When the rod is in static equilibrium, will point P lie to the left of line ℓ (as shown), to the right of line ℓ, or on line ℓ?

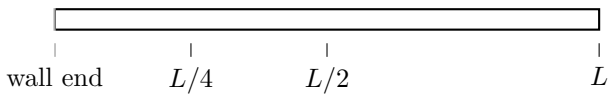
_____ P is to the left of line ℓ _____ P is to the right of line ℓ _____ P is on line ℓ

In a clear, coherent, paragraph-length response that incorporates physics principles, explain your choice.

8a.2. (12 points, suggested time 25 minutes) A uniform shelf board of mass M and length L is held horizontal by two pegs that stick out from a wall. The board does not touch the wall. The upper peg presses on the top surface of the board at the end nearest the wall. A slanted lower peg touches the bottom surface of the board at a single point a distance $L/4$ from that end. A cat of mass m sits at the far end of the board. The pegs exert only vertical forces on the board.



(a) The bar below represents the board. Draw and label the forces (not components) exerted on the board, with each arrow starting at the point where that force is exerted. Represent the cat's effect on the board by a single force.



(b) i. Rank the magnitudes of the following forces: the force F_U exerted by the upper peg, the force F_D exerted by the lower peg, and the weight Mg of the board. If any are equal, indicate that clearly.

Largest _____ _____ _____ Smallest

Briefly justify your ranking.

ii. Derive an expression for the magnitude F_U of the force exerted on the board by the upper peg. Express your answer in terms of M, m, and physical constants, as appropriate.

(c) Derive an expression for the magnitude F_D of the force exerted on the board by the lower peg. Express your answer in terms of M, m, L, and physical constants, as appropriate.

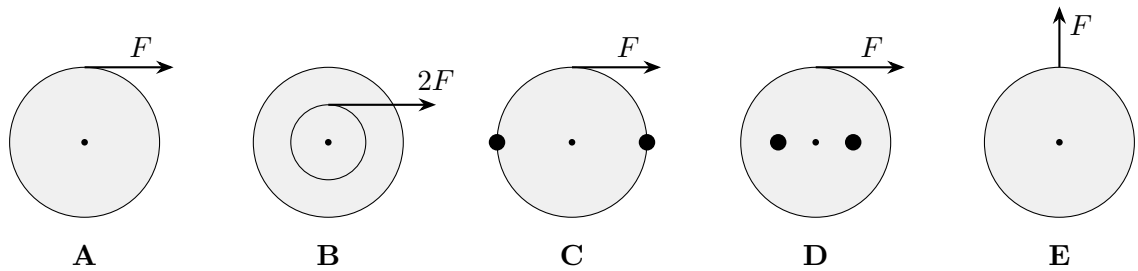
(d) The cat walks toward the wall and sits down at the middle of the board. Does the magnitude F_U of the force exerted by the upper peg increase, decrease, or stay the same?

_____ Increase _____ Decrease _____ Stay the same

Briefly justify your answer.

(e) The cat's mass m is less than the board's mass M. The cat walks all the way to the wall end of the board and sits on the board there, right beside the upper peg, so that the cat's weight acts on the board at the wall end. Derive an expression for the magnitude F_U of the force exerted by the upper peg now. Express your answer in terms of M, m, and physical constants, as appropriate.

8b.1. (7 points, suggested time 13 minutes) Five identical uniform disks, each of mass M and radius R, lie flat on a horizontal table. Each disk can spin without friction about a fixed vertical axle through its center. The rotational inertia of a bare disk about its axle is $\frac{1}{2}MR^2$. Each disk starts at rest, and then a horizontal force is applied to it as shown in the top views below. In C and D, two small lumps of clay, each of mass $\frac{M}{4}$, are stuck to the top surface of the disk at the distances shown. In B, a string is wrapped around a light hub of radius $\frac{R}{2}$ that is fastened to the disk. Treat the clay lumps as particles.



In C, the lumps are at distance R from the axle. In D, they are at distance $\frac{R}{2}$. In E, the force acts at the rim and points directly away from the axle.

(a) Rank the magnitudes of the angular accelerations of the disks, from greatest to least, just after the forces begin to act. If two disks have the same angular acceleration, show that with an equals sign. If a disk has zero angular acceleration, include it in the ranking.

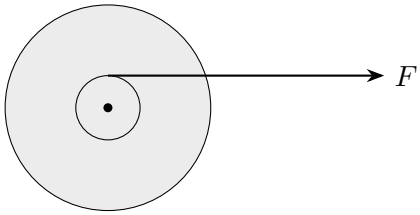
Greatest _____ _____ _____ _____ _____ Least

In a clear, coherent, paragraph-length response that may also contain equations, justify your ranking.

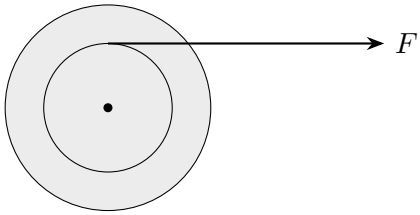
(b) The force on disk D keeps the same magnitude and stays tangent to the rim as the disk turns. Derive an expression for the angular speed of disk D after the force has acted for a time Δt. Express your answer in terms of F, M, R, Δt, and physical constants, as appropriate.

8b.2. (12 points, suggested time 25 minutes) A toy top is mounted in a launcher on a vertical axle, about which the top can spin without friction. The top's rotational inertia about the axle is I. A sleeve of radius r and negligible mass is fastened to the top, and a string of length L is wound around the sleeve. Starting with the top at rest, a student pulls the free end of the string horizontally with a constant force of magnitude F until all of the string has unwound and the string drops away. The string does not slip on the sleeve.

In a second trial, the sleeve is replaced with a sleeve of radius $2r$, also of negligible mass. The same string of length L is wound around it, and the student again pulls with constant force F until the string is fully unwound.



Trial 1: sleeve radius r (top view)



Trial 2: sleeve radius $2r$ (top view)

(a) i. How does the work done by the string on the top in Trial 2 compare with the work done in Trial 1? Without deriving or manipulating equations, explain your answer in terms of the force the string exerts, and also in terms of the torque it exerts on the top.

ii. Is the angular speed of the top when the string drops away in Trial 2 greater than, less than, or the same as in Trial 1?

_____ Greater than _____ Less than _____ The same as

Briefly explain your reasoning without deriving or manipulating equations.

(b) Derive an expression for the angular speed of the top when the string drops away in Trial 1. Express your answer in terms of F, L, I, r, and physical constants, as appropriate.

A student who saw only the setup made this prediction:

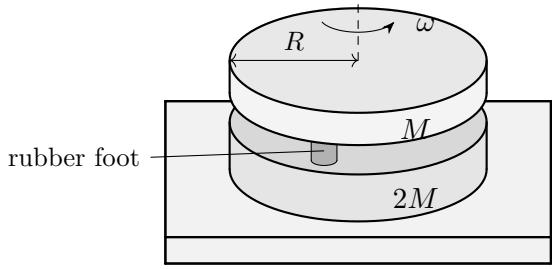
> "With the bigger sleeve, the string pulls with the same force but twice as far from the axle, so the torque on the top is twice as large. The rotational inertia is the same, so the angular acceleration is twice as large. So in Trial 2 the top ends up spinning faster."

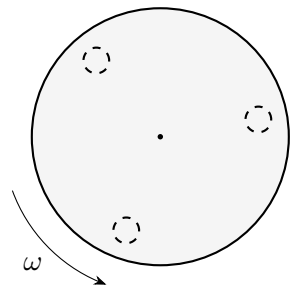
(c) i. Which aspects of the student's reasoning, if any, are correct? Explain how you arrived at your answer.

ii. Which aspects of the student's reasoning, if any, are incorrect? Explain how you arrived at your answer.

(d) Identify a step in your derivation in part (b), other than the final answer, that shows whether the student's conclusion is correct. Explain how that step shows it.

8b.3. (7 points, suggested time 13 minutes) The apparatus in the figure below consists of a horizontal surface, a uniform disk of mass $2M$ resting on the surface, and a second uniform disk of mass M that rests on the first disk on three identical, massless rubber "feet" equally spaced around its rim. Both disks have radius R, and they share a vertical axis. The rotational inertia of a uniform disk of mass M and radius R about its axis is $\frac{1}{2}MR^2$. There is no friction between the horizontal surface and the disk of mass $2M$, but there is friction between the disk of mass $2M$ and the rubber feet. Initially, the disk of mass M is spinning with angular speed ω while the lower disk is at rest. Eventually the two disks spin together with the same angular speed $\omega_{f,1}$.





top view (feet, beneath the upper disk, dashed)

The experiment is repeated, except now with both masses, M and $2M$, increased by the same factor. The initial angular speed is again ω.

(a) How does the final angular speed $\omega_{f,2}$ in the second experiment compare with the final angular speed $\omega_{f,1}$ in the first experiment?

_____ $\omega_{f,2} > \omega_{f,1}$ _____ $\omega_{f,2} < \omega_{f,1}$ _____ $\omega_{f,2} = \omega_{f,1}$

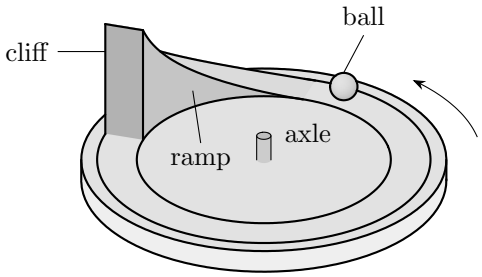
Briefly explain your reasoning.

In each experiment, the rubber feet leave skid marks (arcs) on the lower disk. The skid marks from different feet do not overlap.

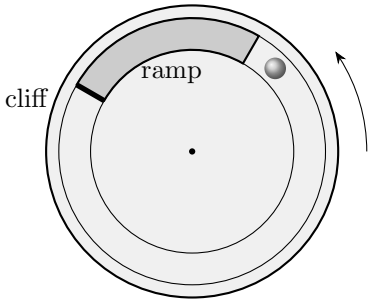
(b) In a clear, coherent, paragraph-length response that incorporates relevant principles of physics, explain whether the skid marks in the second experiment will be longer than, shorter than, or the same length as the skid marks in the first experiment.

(c) Derive an expression for the mechanical energy converted to internal energy by friction at the feet in the first experiment. Express your answer in terms of M, R, ω, and physical constants, as appropriate.

8b.4. (12 points, suggested time 25 minutes) A horizontal disk is mounted on a frictionless, vertical axle about which it can rotate. A thin circular groove carved near the outer edge of the disk holds a small ball that slides in the groove without friction (the ball does not roll). Over part of the circle, a curved ramp is attached to the disk: the groove continues smoothly up the ramp, staying the same distance from the axle, so the ball can slide up the ramp. The far end of the ramp is a vertical "cliff." The rotational inertia of the ball about the axle is I_{BALL}, the rotational inertia of the disk with its ramp is $I_{\text{DISK}} > I_{\text{BALL}}$, and the ball has mass m.



perspective view



top view
arrows: counterclockwise, as seen from above

In a first experiment, the disk is initially at rest and the ball, at the base of the ramp, is moving counterclockwise (as seen from above) along the groove toward the ramp, with angular momentum of magnitude L_1 about the axle. In a second experiment, the ball is initially at rest at the base of the ramp while the disk and ramp rotate clockwise, toward the ball, with angular momentum of magnitude L_1. In each experiment the ball slides partway up the ramp, comes momentarily to rest relative to the ramp at its maximum height, and never reaches the cliff.

(a) Which has the greater angular speed?

_____ The ball at the beginning of the first experiment

_____ The disk at the beginning of the second experiment

Briefly explain your reasoning.

(b) In the first experiment, the disk, which starts at rest, begins to rotate counterclockwise (as seen from above) as the ball slides up the ramp. Explain why, in terms of the force exerted between the ball and the ramp.

(c) Explain how the kinetic energy of the ball–disk system at the instant the ball reaches its maximum height in the first experiment compares with its kinetic energy at that instant in the second experiment.

(d) Is the initial kinetic energy of the ball–disk system in the first experiment greater than, less than, or equal to the initial kinetic energy of the ball–disk system in the second experiment?

_____ Greater than _____ Less than _____ Equal to

Briefly explain your reasoning.

(e) In the first experiment the ball reaches a maximum height h_1 on the ramp, and in the second experiment it reaches a maximum height h_2. How do h_1 and h_2 compare?

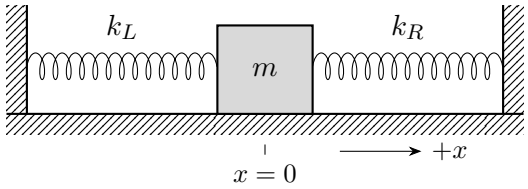
_____ $h_1 > h_2$ _____ $h_1 < h_2$ _____ $h_1 = h_2$

Briefly explain your reasoning.

(f) Derive an expression for h_1. Express your answer in terms of L_1, I_{BALL}, I_{DISK}, m, and physical constants, as appropriate.

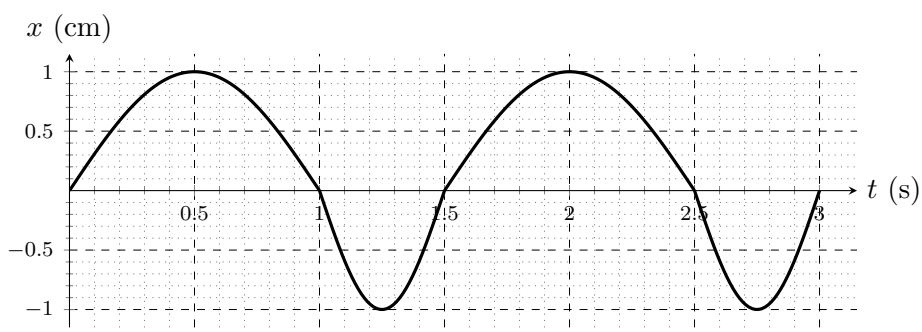
(g) Derive an expression for the ratio h_1/h_2. Express your answer in terms of I_{BALL} and I_{DISK}.

10.1. (7 points, suggested time 13 minutes) In the figure below, a block of mass m is on a frictionless, horizontal floor. The block is in contact with a relaxed massless spring on the left, of spring constant k_L, and in contact with a relaxed massless spring on the right, of spring constant k_R. The other ends of the springs are fixed to immovable walls. The horizontal lengths of the springs can change, but the springs do not bend (the springs remain horizontally oriented). The block is not attached to either spring.



(a) Suppose k_R is much, much greater than k_L. The block is set into oscillatory motion along the x-axis. Derive an algebraic expression for the period of oscillation of the block. Explain how you obtained your result.

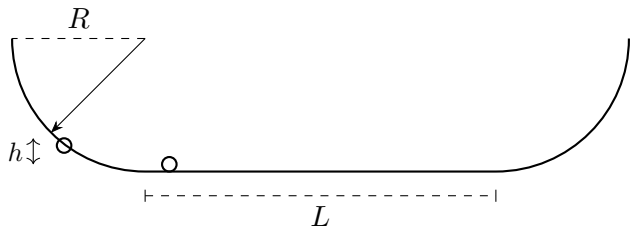
(b) Now suppose only that k_R and k_L are not equal; either spring may be the stiffer one. A student sketches a possible graph of position x vs. time t for the block, shown below.



i. Identify two correct geometric features of the student's sketch and explain, using physics principles, why those features are correct.

ii. Identify two incorrect geometric features of the student's sketch and explain, using physics principles, why those features are incorrect.

10.2. (7 points, suggested time 13 minutes) A frictionless skateboarder's stunt ramp consists of a quarter-circle arc having radius of curvature R connected to a horizontal segment of length L connected to another quarter-circle arc having radius of curvature R. A small ball is released from rest on one of the curved portions of the ramp at a height h above the height that the ball has while on the horizontal segment. The ball slides without rolling. The figure shows two snapshots at different times of the same ball. The height h is much, much smaller than the radius of curvature R.

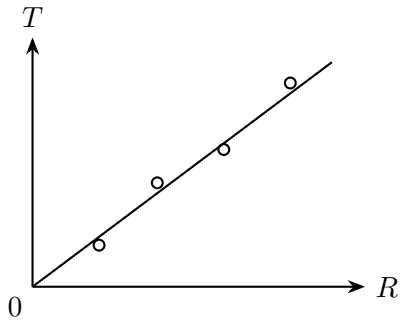


(a) The time it takes for the ball to soonest return to the position from which it is released is T. Would a slight increase in R result in an increase in T, a decrease in T, or no change in T?

_____ Increase in T _____ Decrease in T _____ No change in T

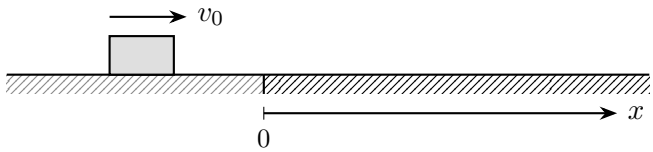
In a clear, coherent, paragraph-length response, explain your reasoning.

On the Internet, a student finds the scatterplot of T vs. R below, labeled as data from a university laboratory experiment with this kind of ramp, using fixed values of h and L. The line is the best-fit line drawn by the website.

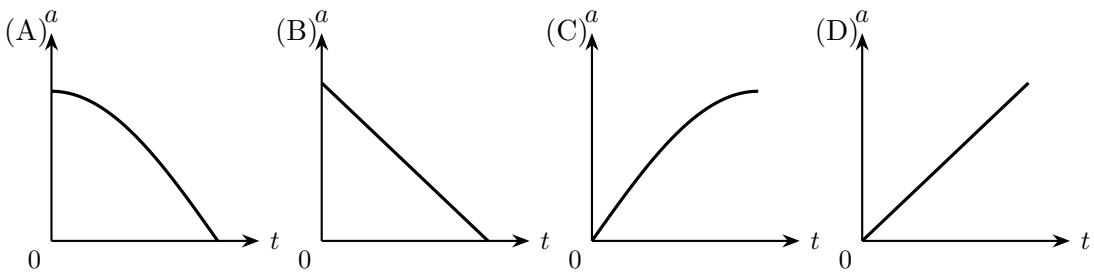


(b) Regardless of whether the scatterplot is correct, identify one feature of the scatterplot that is consistent with your reasoning in part (a) and one feature that is inconsistent with it. Explain each.

10.3. (7 points, suggested time 13 minutes) In the figure below, a block slides to the right with speed v_0 across a frictionless portion of the floor (lighter shading) and then reaches a second portion of the floor (darker shading). In the darker region, the coefficient of kinetic friction μ_K between the block and the floor increases in proportion to the distance x from the line where the two regions meet: $\mu_K = bx$, where b is a positive constant. Even though the block is drawn large, treat it as very small, so that at almost any moment it is entirely on one region or the other.



(a) Which one of the following graphs represents the magnitude of the block's acceleration as a function of time, beginning at the instant the block first reaches the darker region and ending when the block stops?

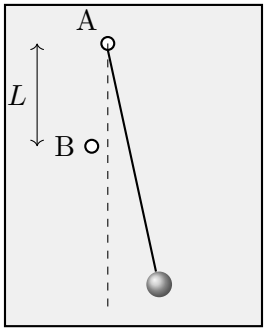


_____ A _____ B _____ C _____ D

In a clear, coherent, paragraph-length response that incorporates physics principles, explain how you arrived at your choice.

(b) Derive an expression for the time the block takes to stop after it first reaches the darker region. Express your answer in terms of b, v_0, and physical constants, as appropriate.

10.4. (7 points, suggested time 13 minutes) In the apparatus shown below, two thin horizontal rods, A and B, stick out toward the viewer from a fixed vertical board; the figure shows the rods end-on, in front of the board. Rod B is a vertical distance L below rod A and slightly to one side. One end of a massless string of length ℓ is tied to rod A, and the other end is tied to a small, massive bob. The bob is released from rest on the side away from rod B. As the bob swings past its lowest position, the string comes into contact with rod B; the bob then rises on the other side and momentarily comes to rest. The bob is always released from a height below rod B, and the angle between any straight segment of the string and the vertical is always small. Neglect the small horizontal offset of rod B when finding times. Let Δt be the time from the release of the bob until it next comes to rest.



vertical board (behind)

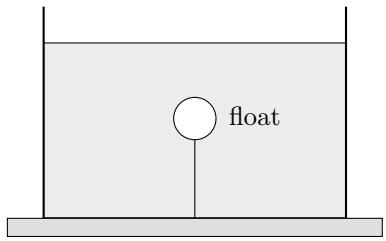
(a) Rod B is moved lower, so that L increases; the string's length ℓ does not change. Does Δt increase, decrease, or stay the same?

_____ Increase _____ Decrease _____ Stay the same

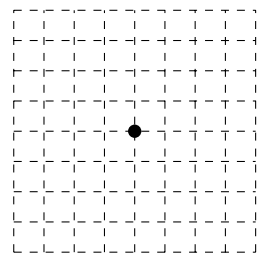
In a clear, coherent, paragraph-length response that incorporates physics principles, explain your reasoning.

(b) Derive an expression for Δt. Express your answer in terms of ℓ, L, and physical constants, as appropriate.

11a.1. (7 points, suggested time 13 minutes) A fish tank sits on a bathroom scale. A small plastic float of volume V and uniform density ρ_f is held completely underwater by a light string tied to the bottom of the tank, as shown. The water has density ρ_w, which is greater than ρ_f. Everything is at rest.



scale



float

(a) On the dot above, which represents the float, draw and label the forces (not components) that act on the float. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Draw the relative lengths of the arrows to show which forces are larger.

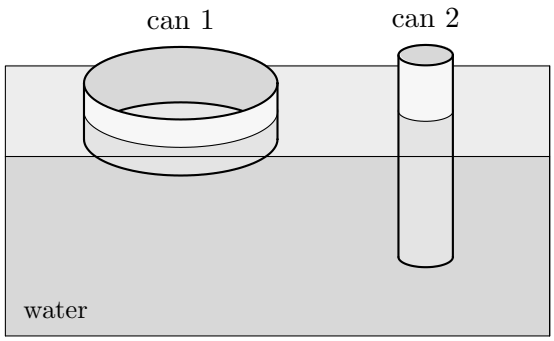
(b) The string is cut. Is the reading of the scale just after the string is cut greater than, less than, or the same as the reading just before the string is cut?

_____ Greater than _____ Less than _____ The same as

In a clear, coherent, paragraph-length response that may also contain diagrams and/or equations, justify your answer.

(c) Derive an expression for the tension in the string before it is cut. Express your answer in terms of V, ρ_f, ρ_w, and physical constants, as appropriate.

11a.2. (7 points, suggested time 13 minutes) Two empty cylindrical aluminum cans are floating upright in water (figure below). Both top caps have been removed. The bottom disk-shaped plate of can 1 is larger than the bottom disk-shaped plate of can 2. An additional mass Δm is deposited into the bottom of can 1, and the same additional mass Δm is also deposited in the bottom of can 2. Care is taken to avoid jostling and overturning the cans. The cans both remain floating, and, after transient oscillations of the cans have damped out, the bottom of can 1 ends up an additional depth d_1 farther below the water surface, and the bottom of can 2 ends up an additional depth d_2 farther below the water surface.



(a) How do d_1 and d_2 compare?

_____ $d_1 > d_2$ _____ $d_1 < d_2$ _____ $d_1 = d_2$

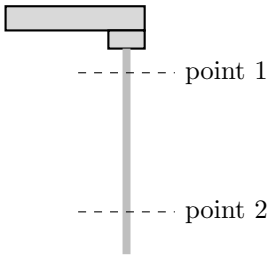
In a clear, coherent, paragraph-length response that incorporates physics principles, explain your reasoning.

(b) Can 1, with the added mass Δm still in it, is moved to a tank of salt water, which is denser than fresh water. Is the depth of the bottom of can 1 below the surface in the salt water greater than, less than, or equal to its depth in the fresh water?

_____ Greater than _____ Less than _____ Equal to

Briefly explain your reasoning.

11b.1. (7 points, suggested time 13 minutes) Water flows steadily and smoothly from a kitchen faucet, forming an unbroken stream that falls straight down. Point 1 is just below the faucet, and point 2 is farther down the stream. (The figure does not show the stream's width to scale.)



Assume the water behaves as an ideal fluid.

(a) Is the cross-sectional area A_2 of the stream at point 2 greater than, less than, or equal to the cross-sectional area A_1 at point 1?

_____ Greater than _____ Less than _____ Equal to

In a clear, coherent, paragraph-length response that incorporates physics principles, explain your reasoning.

(b) The faucet is opened further, so that the water passes point 1 faster than before. The cross-sectional area at point 1 does not change, and points 1 and 2 stay where they are. Is the ratio A_2/A_1 now greater than, less than, or equal to its value before?

_____ Greater than _____ Less than _____ Equal to

Briefly explain your reasoning.

11b.2. (12 points, suggested time 25 minutes) A syringe pump pushes water out of a syringe at a constant volume flow rate Q, which the students want to determine. The pump can be fitted with needles of different inner diameter d. Assume the water behaves as an ideal fluid. For each needle, the students measure the speed v of the water leaving the needle.

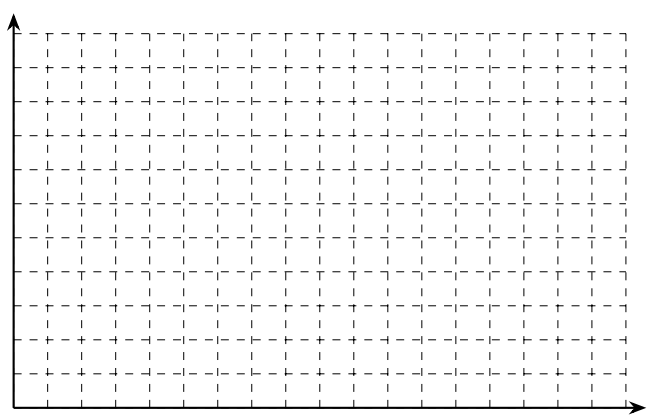
(a) Derive an expression for the speed v of the water leaving a needle of inner diameter d. Express your answer in terms of Q, d, and physical constants, as appropriate.

Inner diameter d (mm)	Exit speed v (m/s)	
0.60	5.41	
0.70	3.82	
0.84	2.79	
1.07	1.62	
1.37	1.03	

(b) Indicate quantities that could be graphed to yield a straight line whose slope could be used to determine Q. You may use the empty column in the table.

Vertical axis: _____ Horizontal axis: _____

(c) On the grid below, plot the quantities you indicated. Clearly scale and label all axes, and draw a best-fit line.

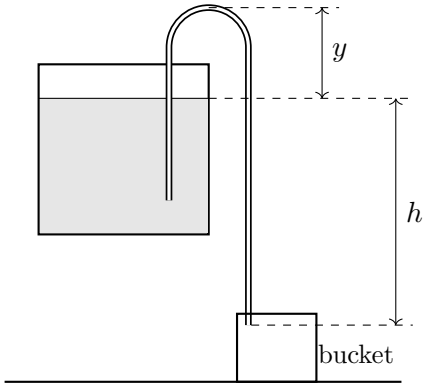


(d) Use your best-fit line to determine the volume flow rate Q.

(e) A student says, "The data show that halving the needle's diameter doubles the speed of the water." Using the data, evaluate the student's statement. If the statement is incorrect, state the relationship between speed and diameter that the data do support, and explain it using physics principles.

AP Physics 1 Chapter 11c: Fluids: statics and flow combined

11c.1. (7 points, suggested time 13 minutes) A flexible hose is used as a siphon to drain water from a large aquarium into a bucket on the floor. The hose is full of water, and water flows steadily out of the open end. The open end of the hose is a vertical distance h below the surface of the water in the aquarium, and the highest point of the hose is a height y above that surface. The hose has a uniform inner diameter much smaller than the width of the aquarium. Assume the water behaves as an ideal fluid.



(a) Derive an expression for the speed v of the water leaving the open end of the hose. Express your answer in terms of h, y, and physical constants, as appropriate.

(b) The hose is lifted so that its highest point is farther above the aquarium's water surface, but the water still flows and the open end stays at the same height. Does the speed of the water leaving the hose increase, decrease, or stay the same?

_____ Increase _____ Decrease _____ Stay the same

Briefly justify your answer.

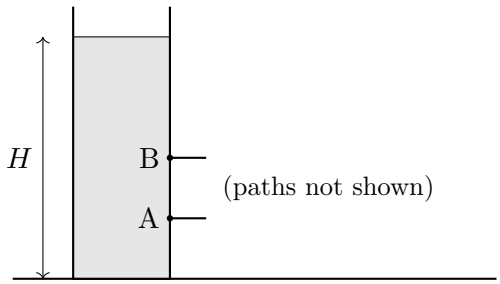
(c) Is the pressure of the water at the highest point of the hose greater than, less than, or equal to atmospheric pressure?

_____ Greater than _____ Less than _____ Equal to

Briefly justify your answer.

(d) The water surface in the aquarium has area A_s, and the inner cross-sectional area of the hose is a. Derive an expression for the speed at which the water surface in the aquarium moves downward while the siphon runs. Express your answer in terms of A_s, a, h, y, and physical constants, as appropriate.

11c.2. (7 points, suggested time 13 minutes) An open-topped plastic bottle stands on a kitchen counter and is kept filled with water to a height H above the counter. Two small holes are poked in the side of the bottle: hole A at a height $H/4$ above the counter and hole B at a height $H/2$ above the counter. Water squirts horizontally out of each hole and lands on the counter. Assume the water behaves as an ideal fluid and that the holes are small compared to the width of the bottle.



A student makes the following claim:

> "The water pressure is greater at hole A because hole A is deeper below the water surface. Greater pressure pushes the water out faster, so the water from hole A lands farther from the bottle than the water from hole B."

(a) i. Identify one aspect of the student's claim that is correct, and briefly explain why it is correct.

ii. Identify one aspect of the student's claim that is incorrect, and explain why it is incorrect.

(b) Derive an expression for the horizontal distance D from the side of the bottle to the point where water from a hole at height y above the counter lands. Express your answer in terms of y, H, and physical constants, as appropriate.

AP Physics 2: Contents

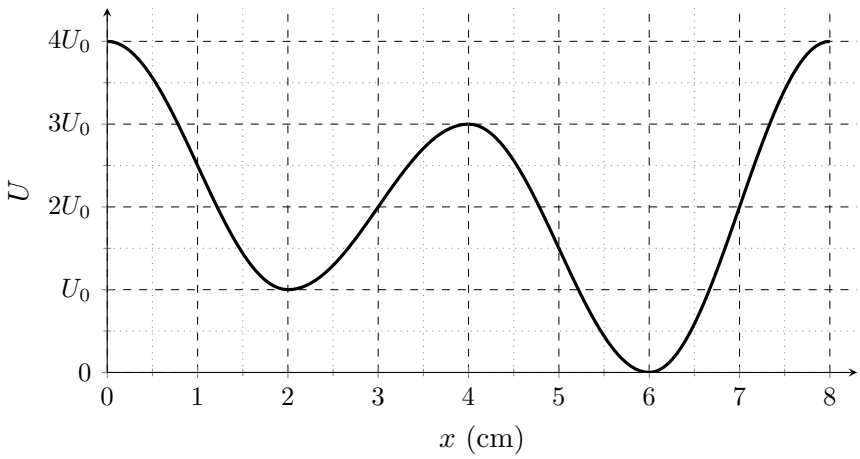
Each cell lists the questions in that chapter that use that format, with page numbers. A question can appear in more than one column, so the format totals add up to more than the number of questions. The Total column counts the questions in each chapter. Question 3.2 is the second question in chapter 3.

Chapter	Total	Calculate or draw	Compare two cases	Rank	Predict a change	Explain without equations	Critique a student	Does it make sense?	Paragraph response	Design an experiment	Analyze data
7 Potential-energy landscapes	2	7.1 (77), 7.2 (79)	7.2 (79)	–	7.1 (77)	7.2 (79)	7.1 (77)	–	–	–	–
11 Waves and sound	4	11.1 (81), 11.2 (83), 11.3 (85), 11.4 (87)	11.2 (83)	–	11.1 (81), 11.2 (83), 11.3 (85), 11.4 (87)	11.2 (83), 11.4 (87)	11.2 (83), 11.3 (85), 11.4 (87)	11.3 (85)	–	–	11.1 (81)
12a Electric charge, field, and potential	2	12a.1 (90), 12a.2 (91)	12a.1 (90)	12a.2 (91)	–	12a.2 (91)	–	12a.2 (91)	–	–	–
12b Circuits and capacitors	2	12b.1 (94), 12b.2 (96)	12b.1 (94)	–	–	12b.1 (94)	–	–	–	12b.2 (96)	12b.2 (96)
13 Magnetism	2	13.1 (98), 13.2 (100)	13.1 (98)	–	13.1 (98), 13.2 (100)	–	13.2 (100)	–	–	–	–
14 Electromagnetic induction	3	14.1 (102), 14.2 (105), 14.3 (107)	14.1 (102), 14.3 (107)	14.2 (105)	14.3 (107)	14.1 (102)	–	–	–	–	–
15 Physical optics	2	15.1 (110), 15.2 (112)	15.2 (112)	–	15.1 (110)	15.2 (112)	–	–	–	–	15.1 (110)
16 Geometric optics	4	16.1 (114), 16.2 (116), 16.3 (118), 16.4 (121)	16.2 (116), 16.3 (118), 16.4 (121)	–	16.1 (114), 16.3 (118), 16.4 (121)	–	16.2 (116)	–	–	–	–
17a Quantum physics	3	17a.1 (124), 17a.2 (126), 17a.3 (129)	–	17a.2 (126)	17a.1 (124)	17a.2 (126), 17a.3 (129)	17a.3 (129)	17a.3 (129)	–	–	17a.1 (124)
17b Subatomic physics	2	17b.1 (132), 17b.2 (135)	17b.1 (132)	–	–	–	17b.2 (135)	–	17b.1 (132)	–	17b.2 (135)

18a Ideal gases and kinetic theory	3	18a.1 (139), 18a.3 (143)	18a.1 (139), 18a.3 (143)	–	18a.2 (141), 18a.3 (143)	18a.1 (139)	18a.3 (143)	18a.3 (143)	–	18a.2 (141)	18a.2 (141)
18b Thermodynamics: first law	3	18b.1 (146), 18b.3 (148), 18b.4 (150)	18b.1 (146), 18b.4 (150)	18b.3 (148)	18b.4 (150)	–	18b.3 (148), 18b.4 (150)	–	–	–	18b.4 (150)
18c Entropy and the second law	2	18c.1 (152), 18c.2 (154)	18c.1 (152)	–	18c.2 (154)	18c.1 (152), 18c.2 (154)	18c.1 (152), 18c.2 (154)	–	–	–	–
18d Supplement: microstates (beyond AP Physics 2)	2	18d.1 (157), 18d.2 (159)	18d.1 (157)	18d.2 (159)	–	18d.1 (157)	18d.2 (159)	–	–	–	–
Total	**36**	**35**	**18**	**5**	**17**	**13**	**14**	**4**	**1**	**2**	**7**

Formats. *Calculate or draw:* derive, calculate, or draw a diagram or graph. *Compare two cases:* say which of two situations has more of something, and why. *Rank:* order several related objects. *Predict a change:* say what happens to a quantity after something is changed. *Explain without equations:* reason in words first, then connect the words to a derivation. *Critique a student:* find what is right and wrong in a student's argument. *Does it make sense?:* test an equation or a graph against physics principles. *Paragraph response:* a clear, coherent, paragraph-length explanation. *Design an experiment:* write a procedure to measure a value or test a claim. *Analyze data:* graph data, often after choosing quantities that make a straight line, and use the slope or intercept.

7.1. (10 points, suggested time 20 minutes) A cart of mass m rolls without friction along a straight horizontal track. Magnets fixed along the track exert forces on a magnet mounted on the cart. The graph shows the potential energy U of the cart–magnets system as a function of the cart's position x along the track. U_0 is a positive constant.



(a) Consider the positions $x = 1$ cm, 3 cm, 5 cm, and 7 cm.

i. At which of these positions is the magnitude of the net force exerted on the cart greatest?

_____ 1 cm _____ 3 cm _____ 5 cm _____ 7 cm

Briefly justify your answer.

ii. What is the direction of the net force exerted on the cart when it is at $x = 3$ cm?

_____ $+x$ _____ $-x$ _____ The force is zero

Briefly justify your answer.

(b) The cart is released from rest at $x = 1$ cm.

i. Will the cart ever reach $x = 6$ cm?

_____ Yes _____ No

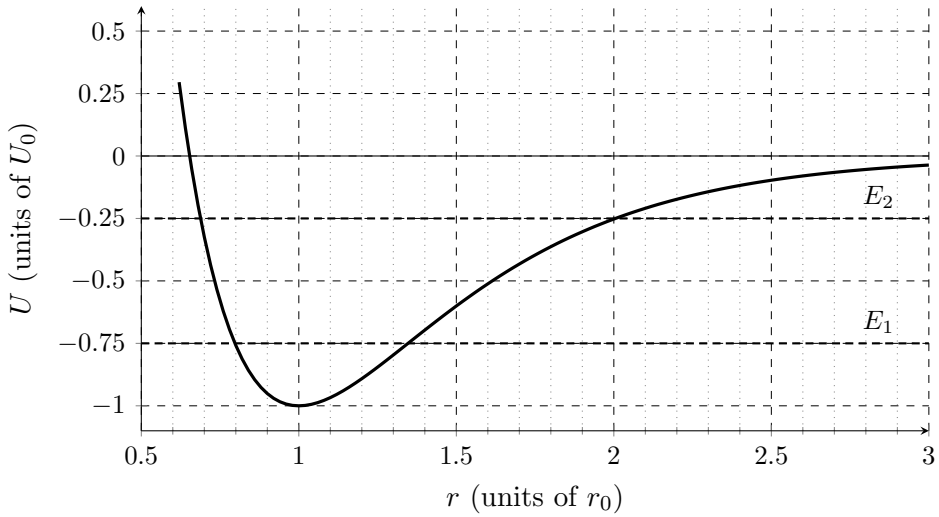
Briefly justify your answer.

ii. Determine the range of positions the cart moves through. Explain how you used the graph to determine it.

(c) A student says, "At $x = 2$ cm the potential energy is at a minimum, so the cart must be momentarily at rest there." Explain what is wrong with the student's statement.

(d) The cart is now launched from $x = 2$ cm toward $+x$. Derive an expression for the least speed it can be given at $x = 2$ cm so that it reaches $x = 6$ cm. Then suppose it is launched with slightly more than that least speed. Derive an expression for its approximate speed at $x = 6$ cm. Express your answers in terms of m, U_0, and physical constants, as appropriate.

7.2. (12 points, suggested time 25 minutes) Two atoms that are neighbors in a solid can be modeled as a pair that vibrates along the line joining them. The graph shows the potential energy U of the two-atom system as a function of the separation r between the atoms' centers. Separation is given in units of r_0, and energy in units of U_0, where r_0 and U_0 are positive constants. The horizontal dashed lines show two possible values, E_1 and E_2, of the total energy of the vibrating pair.



(a) i. What is the separation at which the net force each atom exerts on the other is zero? Briefly justify your answer.

ii. When the separation is $0.75r_0$, is the force between the atoms attractive or repulsive?

_____ Attractive _____ Repulsive

Briefly justify your answer.

(b) i. For total energy E_1, estimate the smallest and largest separations reached as the pair vibrates.

ii. Repeat for total energy E_2.

iii. For which total energy is the average of the smallest and largest separations greater?

_____ E_1 _____ E_2 _____ The same for both

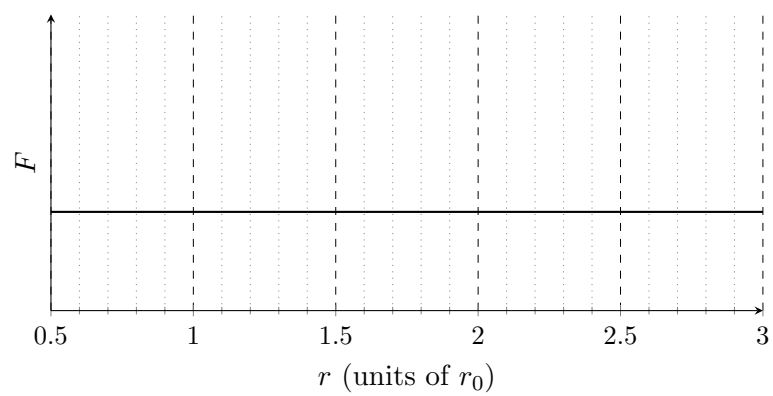
Support your answer by calculating the average for each total energy from your estimates.

iv. For total energy E_2, determine the greatest kinetic energy of the pair as it vibrates, and the separation at which it occurs.

(c) Most solids expand when they are heated. First consider a pair of atoms whose potential-energy graph had the same shape on both sides of its minimum. Explain why heating such pairs would not change the average separation of the atoms.

(d) Using features of the graph for the actual pair, explain how this model accounts for the expansion of solids when they are heated.

(e) i. On the axes below, sketch the force F that each atom exerts on the other as a function of the separation r, for separations from $0.6r_0$ to $3r_0$. Take F to be positive when the force is repulsive. No numerical scale is needed on the F axis.



ii. Use the graph of U to estimate the magnitude of the greatest attractive force between the atoms. Express your answer in terms of U_0 and r_0, and show how you used the graph.

11.1. (12 points, suggested time 25 minutes) Students study the vibration of a guitar string. The string is fixed at both ends, and the vibrating length between the fixed points is $L = 0.65$ m. One end of the string is attached to a luggage scale, which reads the tension T in the string; the students change the tension by turning a peg. For each tension, the students pluck the string and use a tuner to measure the frequency f of the fundamental (lowest) standing-wave mode.

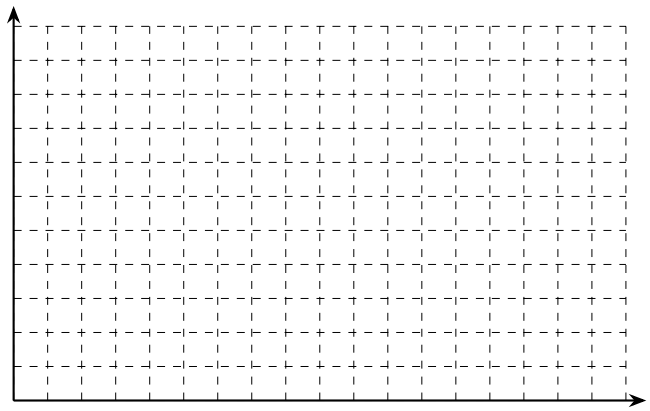
(a) Derive an expression for the fundamental frequency f in terms of T, L, the string's mass per unit length μ, and physical constants, as appropriate.

Tension T (N)	Fundamental frequency f (Hz)	
40	141	
60	171	
80	199	
100	221	
120	244	

(b) Indicate quantities that could be graphed to yield a straight line whose slope could be used to determine μ. You may use the empty column in the table.

Vertical axis: _____ Horizontal axis: _____

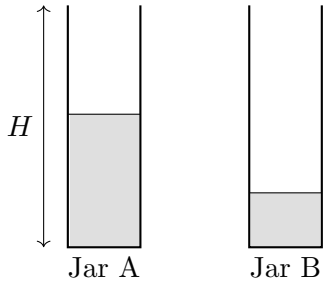
(c) On the grid below, plot the quantities you indicated. Clearly scale and label all axes, and draw a best-fit line.



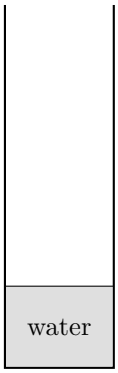
(d) Use your best-fit line to determine μ.

(e) The string is replaced with a string that has four times the mass per unit length, with the same vibrating length L, and the tension is set to 80 N. Predict the fundamental frequency of the new string. Briefly justify your prediction.

11.2. (10 points, suggested time 20 minutes) Two identical tall cylindrical glass jars, each of height H, are open at the top. Jar A contains more water than jar B. When a student blows across the top of each jar, the air column above the water vibrates in its fundamental mode and the jar produces a tone.



(a) i. The figure below is an enlarged copy of jar B, with the same proportions. On the figure, sketch the pattern of the air's displacement amplitude along the air column when it vibrates in its fundamental mode. Label each displacement node N and each displacement antinode A.



Jar B (enlarged)

ii. Is the frequency of the tone from jar A higher than, lower than, or the same as the frequency of the tone from jar B?

_____ Higher than _____ Lower than _____ The same as

Briefly justify your answer.

(b) Derive an expression for the fundamental frequency f of a jar in which the water has depth d. Express your answer in terms of H, d, the speed of sound v_s, and physical constants, as appropriate.

(c) With water of depth $H/4$ in a jar, the tone has frequency f_1. Water is then added until the depth is $H/2$.

i. Use your expression from part (b) to derive an expression for the new frequency in terms of f_1.

ii. A student says, "Doubling the depth of the water changes the frequency by a factor of 2." Is the student correct?

_____ Yes _____ No

Briefly explain your reasoning.

(d) The jars are taken into a warmer room, where the speed of sound in air is greater. The water depths do not change. Does the frequency of the tone from jar B increase, decrease, or stay the same?

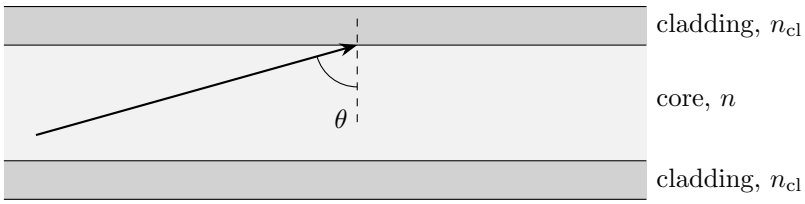
_____ Increase _____ Decrease _____ Stay the same

Briefly justify your answer.

(e) When the student blows harder across jar B, a higher tone is heard, produced by the next standing-wave mode of the air column above the fundamental. Determine the ratio of the frequency of this tone to the fundamental frequency of jar B. Briefly explain how you determined it.

11.3. (12 points, suggested time 25 minutes) Two musicians play together over the internet. The sound of each musician's instrument is converted to a digital signal that travels as pulses of light a length L in a glass optical fiber to the other musician. Musicians who play together in one room already cope with a delay: sound travels at speed v_s in air, and players a few meters apart routinely play in time. Light in the fiber's core travels at speed c/n, where n is the index of refraction of the core. Treat the signal as moving along the fiber at speed c/n. (Real connections add further delays in audio interfaces, computer processing, and the internet links into each neighborhood and home. Ignore all of these delays in this question.)

The figure shows a section of the fiber cut along its length. The core, of index n, is surrounded by a layer of glass called the cladding, of index n_{cl}. A ray of light in the core strikes the boundary between the core and the cladding at an angle θ from the normal.



(not to scale)

(a) i. For light in the core to be totally reflected at the boundary, must n_{cl} be greater than, less than, or equal to n?

_____ Greater than _____ Less than _____ Equal to

Briefly justify your answer.

ii. Derive an expression for the smallest angle θ for which the ray is totally reflected at the boundary. Express your answer in terms of n, n_{cl}, and physical constants, as appropriate.

(b) Two musicians in one room stand a distance d apart. Derive an expression for the distance d for which the time for sound to travel from one musician to the other equals the time for the signal to travel the length L in the fiber. Express your answer in terms of L, n, v_s, and physical constants, as appropriate.

(c) Some newer fibers guide light through a hollow core filled with air, with an index of refraction very close to 1. (These fibers guide light by a mechanism other than total internal reflection.) If the signal instead traveled the same length L in a hollow-core fiber, would the distance d of part (b) be greater than, less than, or the same as with the glass-core fiber?

_____ Greater than _____ Less than _____ The same as

Briefly justify your answer.

(d) Musician A plays a note. Musician B plays a note as soon as she hears A's note. A student says:

> "Musician A hears B's note only after the signal has traveled the length of the fiber twice, so the delay A notices is twice as long. To give A the same delay in one room, the musicians would have to stand $2d$ apart."

Explain what is wrong with the student's conclusion, and state the correct separation in one room.

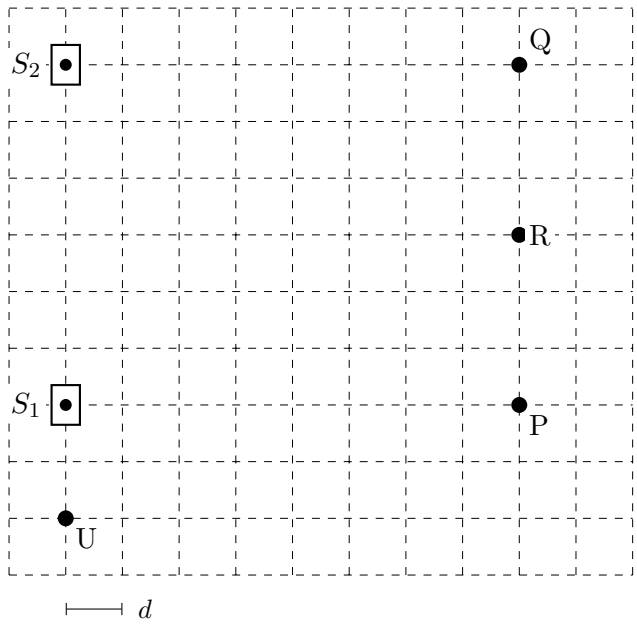
(e) The signal travels 1,200 km in the fiber between the two musicians' cities. (Fiber is laid along roads and rail lines, so this length is greater than the straight-line distance between the cities, which here is about 950 km.) The core of the fiber has index of refraction $n = 1.47$, and the speed of sound in the room is 343 m/s. Calculate the time for the signal to travel this length in the fiber, and calculate the separation d in one room that gives the same delay. Then calculate the length in the fiber that gives the same delay as a separation of 8.0 m in one room.

(f) Instead of the fiber, the musicians could send the signal through a communications satellite in geostationary orbit, about 36,000 km above Earth's surface. The signal travels as radio waves at speed c from one musician up to the satellite and from the satellite down to the other musician. Would the time for the signal to travel from one musician to the other be greater than, less than, or the same as the time you calculated in part (e)?

_____ Greater than _____ Less than _____ The same as

Briefly justify your answer.

11.4. (12 points, suggested time 25 minutes) Two identical loudspeakers, S_1 and S_2, stand on the stage of an empty theater. One tone generator drives both, so they emit sound of the same single frequency, in phase with each other. The theater's walls and seats absorb sound well, so reflections can be neglected. The top-view map below is drawn on a grid of squares of side d and shows four points, P, Q, R, and U, at ear height. Each speaker is centered on a grid point, and all distances are measured from those points.



(a) A listener standing at point R hears a loud tone. Briefly explain why R is a point of constructive interference.

(b) The listener walks in a straight line from R to P. The tone gets steadily quieter, and P is the first point on the walk where it is quietest. Derive an expression for the wavelength of the sound. Express your answer in terms of d.

(c) Is point U a point of constructive interference, destructive interference, or neither?

_____ Constructive _____ Destructive _____ Neither

Briefly explain your reasoning.

While the listener stands at P, a student says:

> "If we unplug speaker S_2, the speakers will send out half as much sound energy as before, so the tone at P will get quieter."

(d) i. Identify one aspect of the student's statement that is correct, and briefly explain why it is correct.

ii. Identify one aspect of the student's statement that is incorrect, and explain why it is incorrect.

(e) With both speakers connected, the frequency of the tone generator is doubled. Is the tone at P now louder than, quieter than, or about as loud as it was before the change?

_____ Louder than _____ Quieter than _____ About as loud as

Briefly explain your reasoning.

The tone generator is set back to its original frequency, and speaker S_2 is unplugged, so only S_1 makes sound. A side door of the theater, about $8d$ wide, opens onto a hallway whose walls also absorb sound well. A person standing in the hallway, off to one side of the doorway, cannot see speaker S_1 but can hear the tone.

(f) The frequency of the tone generator is then lowered. Does the sound that passes through the doorway spread over a wider range of directions, a narrower range, or the same range as before?

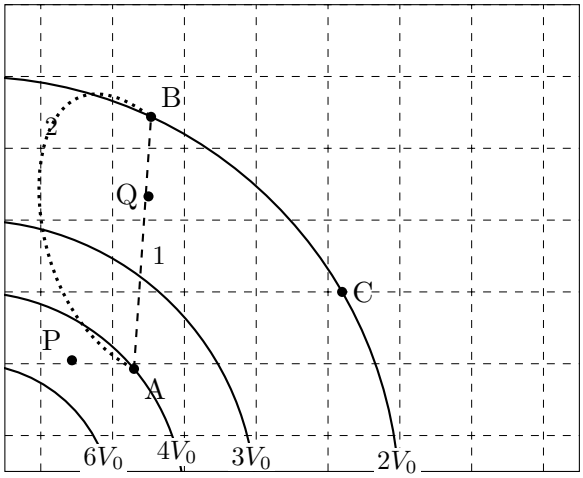
_____ Wider _____ Narrower _____ The same

Briefly explain your reasoning.

(g) Light and sound are both waves, and both pass through the same doorway. Explain why the person in the hallway can hear speaker S_1 but cannot see it.

AP Physics 2 Chapter 12a: Electric charge, field, and potential

12a.1. (10 points, suggested time 20 minutes) The figure shows a map of equipotential lines in a region of space where there is an electrostatic field. The charge that produces the field is outside the mapped region. The lines are labeled with their potentials in terms of a positive constant V_0, and each grid square represents 1 cm by 1 cm. Points P, Q, A, B, and C are marked. Path 1 and path 2 both go from A to B.



(a) On the map, draw an arrow at point P and an arrow at point Q to represent the electric field at those points. Draw the relative lengths of the arrows to show which field is stronger.

(b) A hand slowly moves a small object of charge $+q$ from A to B along path 1. It then returns the object to A and moves it from A to B along path 2.

i. Is the work done by the hand along path 2 greater than, less than, or the same as the work done by the hand along path 1?

_____ Greater than _____ Less than _____ The same as

Briefly justify your answer.

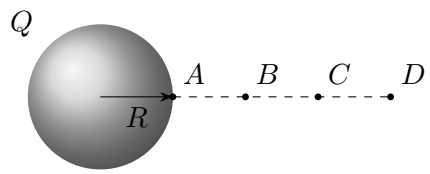
ii. Derive an expression for the work done by the hand on the object as it is moved from A to B along path 1. Express your answer in terms of q, V_0, and physical constants, as appropriate.

(c) An electron (charge $-e$, mass m) is released from rest at point C. Only the electric force acts on it.

i. On the map, draw an arrow at C showing the direction in which the electron begins to move. Briefly justify your answer.

ii. Derive an expression for the speed of the electron when it reaches the line labeled $4V_0$. Express your answer in terms of e, m, V_0, and physical constants, as appropriate.

12a.2. (10 points, suggested time 20 minutes) A conducting sphere of radius R has positive charge Q. Point A is located on the surface of the sphere, point B is located at a distance $2R$ from the center of the sphere, point C is located at a distance $3R$ from the center of the sphere, and point D is located at a distance $4R$ from the center of the sphere, with the center of the sphere and points A, B, C, and D all lying on a single line. The electric potential V_A at position A is greater than the electric potential V_D at position D by an amount $\Delta V_{D\to A}$. Region I is between points A and B, region II is between points B and C, and region III is between points C and D.



(a) Suppose it could be arranged for the electric field strengths only in region I to be doubled, or, instead, for the electric field strengths only in region II to be doubled, or, instead, for the electric field strengths only in region III to be doubled.

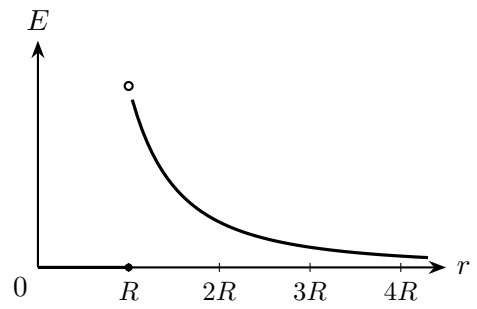
i. Explain how the potential difference across one of the regions depends on the electric field strengths within that region, and how that potential difference changes if all the field strengths within the region are doubled.

ii. In which one of the three regions should the electric field strengths be doubled to increase the potential difference $\Delta V_{D\to A}$ between positions D and A by the greatest amount?

_____ Region I only _____ Region II only _____ Region III only

Briefly explain your reasoning.

The apparatus is kept in its original configuration: none of the modifications offered in part (a) has been applied. On the Internet, a student finds the following sketch of the strength of the electric field as a function of distance r from the center of a conducting sphere.

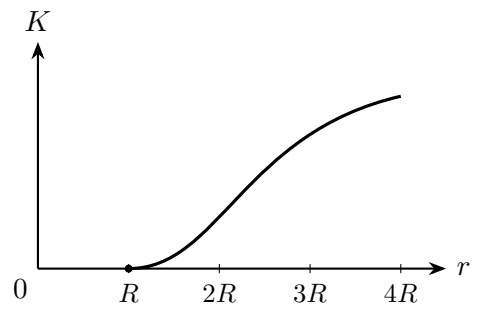


(b) Regardless of whether the sketch is actually correct, is the sketch consistent with your reasoning in part (a)?

_____ Consistent _____ Inconsistent

Briefly explain your reasoning.

On the Internet, the student also finds the following sketch of the kinetic energy K of a positively charged particle released from rest just outside the conducting sphere, as a function of the particle's distance r from the center.



(c) Regardless of whether the sketch is actually correct, is the sketch consistent with your reasoning in part (a)?

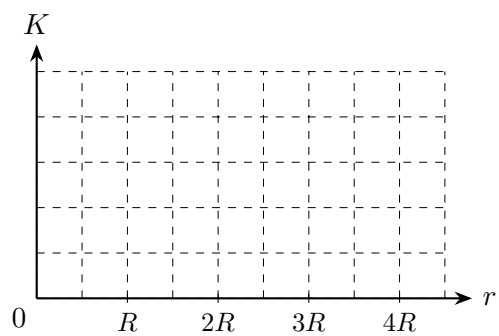
_____ Consistent _____ Inconsistent

Briefly explain your reasoning.

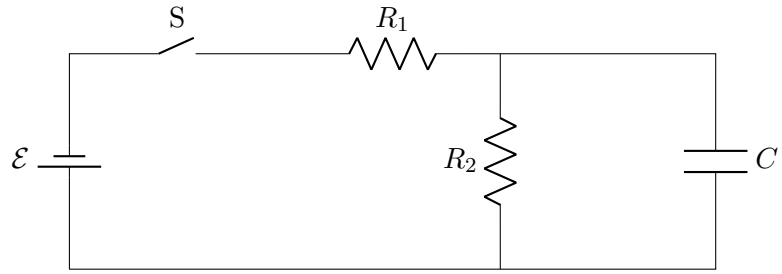
The particle in part (c) has positive charge q and is released from rest just outside the sphere, at $r = R$. The sphere is held fixed, and gravity is negligible.

(d) Derive expressions for the particle's kinetic energy K_B as it passes point B and K_D as it passes point D. Express your answers in terms of q, Q, R, and physical constants, as appropriate. Then determine what fraction of K_D the particle gains while crossing region I.

(e) On the axes below, sketch the particle's kinetic energy K as a function of r from $r = R$ to $r = 4R$. Your sketch should be consistent with your answers to part (d).



12b.1. (12 points, suggested time 25 minutes) In the circuit below, an ideal battery of emf $\mathcal{E}$ is connected in series with a switch S and a resistor of resistance R_1. The other end of R_1 connects to a resistor of resistance R_2 and a capacitor of capacitance C, which are connected in parallel with each other. The capacitor is initially uncharged. At time $t = 0$, the switch is closed.



(a) Is the current in resistor R_1 just after the switch is closed greater than, less than, or the same as the current in R_1 a long time after the switch is closed?

_____ Greater than _____ Less than _____ The same as

Briefly justify your answer.

(b) Answer the following without deriving or manipulating equations.

i. Explain why, a long time after the switch is closed, the potential difference across the capacitor is less than $\mathcal{E}$.

ii. The circuit is rebuilt with the resistor R_2 replaced by one of resistance $2R_2$. Nothing else is changed. Is the charge on the capacitor a long time after the switch is closed greater than, less than, or the same as it was in the original circuit?

_____ Greater than _____ Less than _____ The same as

Briefly explain your reasoning.

(c) For the original circuit, derive an expression for the charge on the capacitor a long time after the switch is closed. Express your answer in terms of $\mathcal{E}$, R_1, R_2, C, and physical constants, as appropriate.

(d) Identify the equation or step in your derivation in part (c) that supports your reasoning in part (b)(ii). Do not simply restate your final answer from part (c). Explain how that equation or step supports your reasoning.

(e) On the axes below, sketch the current I_2 in resistor R_2 as a function of time t, from $t = 0$ until a long time after the switch is closed. Label any values of current on the vertical axis that your sketch reaches or approaches.

12b.2. (12 points, suggested time 25 minutes) Conductive dough is a soft, salty modeling dough that conducts electric current. A student rolls one lump of the dough into a uniform cylinder, presses a thin metal foil disk against each end, and measures the resistance between the disks. The student then makes the claim below.

> "Resistance is proportional to length. So if I roll this same lump into a cylinder twice as long, its resistance will double."

Available equipment: the lump of dough (volume V_d), a smooth tabletop, two foil disks, a meterstick, calipers, a multimeter that can measure resistance, current, or potential difference, and connecting wires.

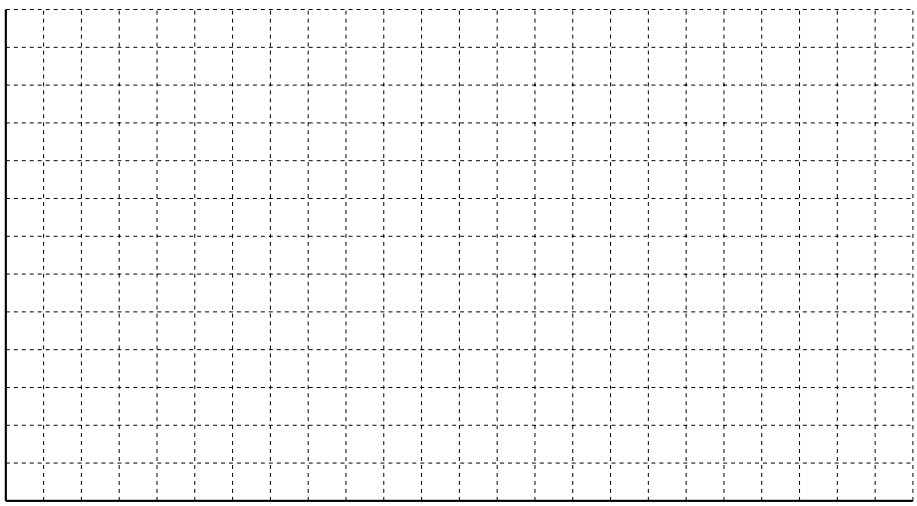
(a) Describe an experimental procedure the student could use to collect data to test the claim. Give enough detail that another student could carry out your procedure, and state which equipment is used to measure each quantity.

(b) Assume the dough is ohmic with resistivity ρ that does not depend on its shape. Derive an expression for the resistance R of a uniform cylinder made from the whole lump, in terms of its length ℓ, V_d, ρ, and physical constants, as appropriate.

The student rolls the lump, of volume $V_d = 8.0 \times 10^{-6}$ m^3, into five cylinders of different lengths and records the data below.

Trial	Length ℓ (m)	Resistance R (Ω)
1	0.050	82
2	0.100	305
3	0.150	716
4	0.200	1238
5	0.250	1970

(c) Choose quantities to graph that should give a straight line if your expression from part (b) is correct. Use the empty columns in the table if helpful. On the grid below, plot the data, label the axes with quantities and units, choose scales, and draw a best-fit line.



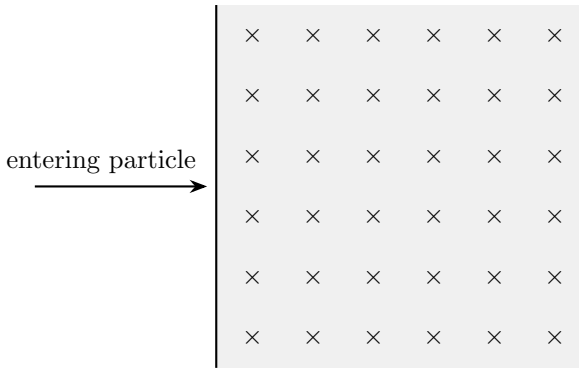
(d) Do the data support the student's claim?

_____ Yes _____ No

Briefly justify your answer by referring to the data.

(e) Use your best-fit line to determine the resistivity of the dough.

13.1. (10 points, suggested time 20 minutes) In a proton-therapy facility, charged particles that start essentially at rest are accelerated through a potential difference V and then enter a region of uniform magnetic field of magnitude B, directed into the page. Each particle enters the field region moving perpendicular to the field and to the region's straight boundary, and later leaves the field region. The figure is not to scale; the field region is large enough to contain each path. A proton has charge $+e$ and mass m. An alpha particle has charge $+2e$ and mass $4m$.



(a) On the figure above, sketch the path of a proton inside the field region.

(b) Derive an expression for the radius r of the proton's path. Express your answer in terms of m, e, V, B, and physical constants, as appropriate.

(c) An alpha particle is accelerated through the same potential difference V and enters the same field. Is the radius of the alpha particle's path greater than, less than, or the same as the radius of the proton's path?

_____ Greater than _____ Less than _____ The same as

Briefly justify your answer.

(d) The potential difference is increased, so protons enter the field faster. Does the time a proton spends in the field region increase, decrease, or stay the same?

_____ Increase _____ Decrease _____ Stay the same

Briefly explain your reasoning.

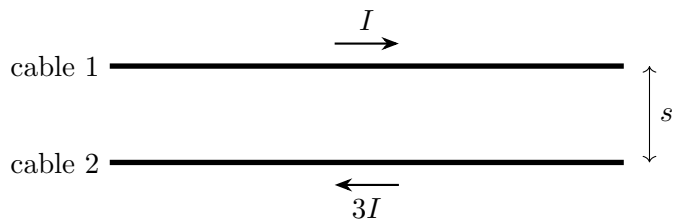
(e) In one treatment setting, $V = 2.0 \times 10^6$ V and $B = 1.5$ T. The mass of a proton is 1.67×10^{-27} kg. Calculate the following.

i. The speed of a proton as it enters the field region

ii. The distance between the point where the proton enters the field region and the point where it leaves

iii. The time the proton spends in the field region

13.2. (12 points, suggested time 25 minutes) Two long, straight single-conductor cables lie parallel to each other on a workshop bench, a distance s apart. Each cable is part of a different direct-current circuit, and the rest of each circuit is far away. Cable 1 carries a steady current I toward the right, and cable 2 carries a steady current $3I$ toward the left, as shown in the top view below. The cables are much longer than s. Ignore the Earth's magnetic field.



(top view)

(a) What is the direction of the magnetic field produced by cable 1 at the location of cable 2?

_____ Toward the top of the page _____ Toward the bottom of the page _____ Into the page _____ Out of the page

(b) Do the cables attract each other, repel each other, or exert no magnetic force on each other?

_____ Attract _____ Repel _____ No magnetic force

Briefly justify your answer.

A student makes the following claim:

> "Cable 2 carries three times as much current as cable 1, and the magnetic force on a current-carrying wire is proportional to the current. So the magnetic force that cable 2 exerts on cable 1 is three times as large as the magnetic force that cable 1 exerts on cable 2."

(c) i. Identify one aspect of the student's claim that is correct, and briefly explain why it is correct.

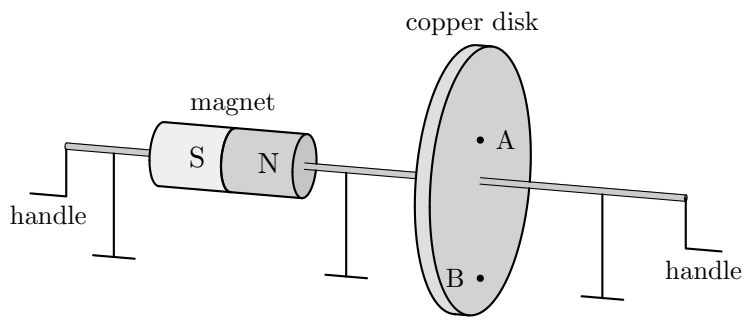
ii. Identify one aspect of the student's claim that is incorrect, and explain why it is incorrect.

(d) Derive an expression for the magnitude of the magnetic force exerted on a length ℓ of cable 1 by cable 2. Express your answer in terms of I, s, ℓ, and physical constants, as appropriate.

(e) The cables are moved apart so that the distance between them is $2s$, with the same currents. By what factor does the magnitude of the force from part (d) change? Briefly justify your answer.

(f) The cables are returned to a distance s apart, with the same currents. Point P is on the bench midway between the two cables. Derive an expression for the magnitude of the net magnetic field at point P, and state its direction. Express your answer in terms of I, s, and physical constants, as appropriate.

14.1. (10 points, suggested time 20 minutes) The apparatus below consists of a cylindrical bar magnet, magnetized along its length, and a copper disk, mounted on a common axis. The magnet and, separately, the disk can be spun about that axis using the attached handles. The supporting structures, including the rods through the magnet and the disk, are made of an electrically insulating material. Two dots are inked onto the copper disk, one at position A (near the center of the disk) and another at position B (farther out from the center of the disk).



For parts (a)–(e), the magnet is held still and the disk spins with constant angular speed ω, counterclockwise as viewed from the handle at the right. Model the magnet's field throughout the disk as uniform, with magnitude B, pointing along the axis away from the magnet.

(a) Consider a free electron in the copper at a distance r from the axis. Derive an expression for the magnitude of the magnetic force exerted on it, in terms of e, ω, r, B, and physical constants, as appropriate. Determine whether this force is directed toward or away from the axis.

(b) The free electrons move along the radius until the charge they rearrange produces an electric field whose force on each free electron balances the magnetic force. Derive an expression for the magnitude of this electric field at a distance r from the axis, in terms of ω, r, B, and physical constants, as appropriate. Determine whether the field points toward or away from the axis.

(c) Dots A and B are at distances r_A and r_B from the axis. The disk's angular speed is doubled, with everything else unchanged. Is the magnitude of the potential difference between dots A and B now greater than twice, less than twice, or equal to twice its value at angular speed ω?

_____ Greater than twice _____ Less than twice _____ Equal to twice

Briefly explain your reasoning.

(d) Which dot is at the higher electric potential?

_____ Dot A _____ Dot B _____ Neither; they are at the same potential

Briefly explain your reasoning.

(e) A third dot, C, is inked on the disk at a distance $\frac{1}{2}(r_A + r_B)$ from the axis, halfway between the distances of A and B. Is the magnitude of the potential difference between dots A and C greater than, less than, or equal to the magnitude of the potential difference between dots C and B?

_____ Greater than _____ Less than _____ Equal to

Briefly explain your reasoning.

Now the magnet and the disk can each be either stationary (not spinning) or in the midst of spinning with constant angular velocity.

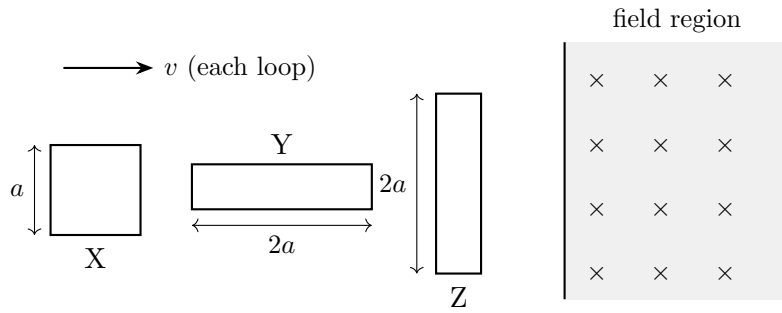
(f) For each of the scenarios in the table below, indicate whether a potential difference is being sustained between inked positions A and B on the copper disk.

Magnet	**Disk**	**Potential difference between dots A and B**
Still	Still	_____ Zero _____ Nonzero
Still	Spinning	_____ Zero _____ Nonzero
Spinning	Still	_____ Zero _____ Nonzero
Spinning	Spinning	_____ Zero _____ Nonzero

i. Briefly explain your choice for the scenario in which the magnet spins and the disk is still.

ii. Briefly explain your choice for the scenario in which both the magnet and the disk spin.

14.2. (12 points, suggested time 25 minutes) Three loops are bent from the same kind of wire, which has resistance λ per unit length. Loop X is a square of side a. Loop Y is a rectangle $2a$ long and $a/2$ high. Loop Z is a rectangle $a/2$ long and $2a$ high. Each loop is pulled to the right at the same constant speed v into a region of uniform magnetic field of magnitude B directed into the page. The figure shows the loops before they reach the field region; the loops are pulled one at a time. The field region is large enough to contain each loop entirely.



(a) Rank the magnitudes of the emfs $\mathcal{E}_X$, $\mathcal{E}_Y$, $\mathcal{E}_Z$ induced in the loops while each loop is partly inside the field region. If any are equal, indicate that clearly.

Largest _____ _____ _____ Smallest

Briefly justify your ranking.

(b) Derive expressions for the magnitudes of the currents I_X, I_Y, and I_Z in the loops while each loop is partly inside the field region. Express your answers in terms of v, B, λ, and physical constants, as appropriate.

(c) Derive an expression for the magnitude of the force that must be exerted on loop X to keep it moving at constant speed while it is partly inside the field region. Express your answer in terms of a, v, B, λ, and physical constants, as appropriate.

(d) i. While loop X is partly inside the field region, is the induced current in loop X clockwise or counterclockwise?

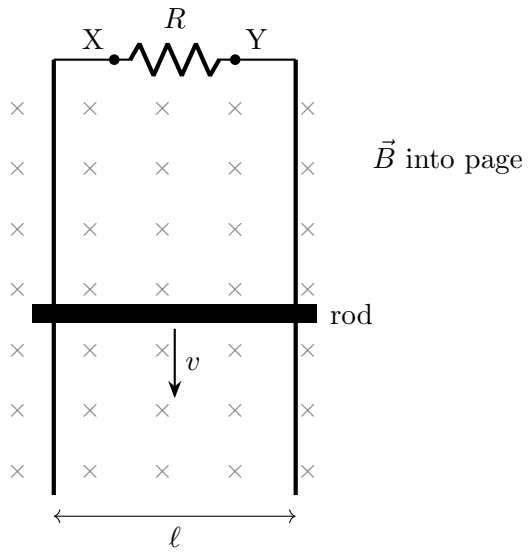
_____ Clockwise _____ Counterclockwise

Briefly justify your answer.

ii. Loop X keeps moving at speed v until it is entirely inside the field region. What is the current in loop X then? Briefly justify your answer.

(e) Each loop is pulled at speed v from the moment its leading edge reaches the field region until it is entirely inside the field region. Derive an expression for the total energy dissipated in loop Z during this time. Express your answer in terms of a, v, B, λ, and physical constants, as appropriate.

14.3. (10 points, suggested time 20 minutes) Two long, vertical metal rails a distance ℓ apart are connected at the top by a resistor of resistance R. A horizontal metal rod of mass m rests across the rails. The rod can slide up and down along the rails without friction, staying horizontal and in contact with both rails. A uniform magnetic field of magnitude B is directed into the page throughout the region shown. Points X and Y are on the wires at the two ends of the resistor. The rails and the rod have negligible resistance, and air resistance is negligible. The rod is released from rest and falls.



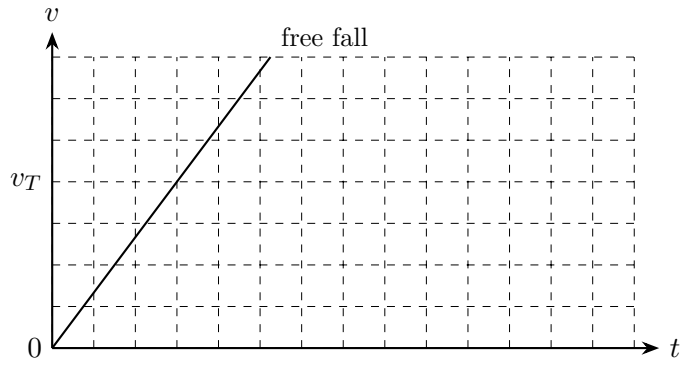
(a) While the rod is falling, is the current in the resistor directed from X to Y or from Y to X?

_____ From X to Y _____ From Y to X

Briefly justify your answer.

(b) The rails are long enough that the rod eventually falls at a constant speed v_T. Derive an expression for v_T. Express your answer in terms of m, ℓ, B, R, and physical constants, as appropriate.

(c) On the axes below, the solid straight line shows the speed of an object that falls freely from rest. Sketch the speed v of the rod as a function of time t, starting from the moment it is released.



(d) The rod has mass 0.020 kg, the rails are 0.50 m apart, $B = 0.80$ T, and $R = 2.0\ \Omega$. Calculate the rate at which energy is dissipated in the resistor while the rod falls at constant speed. Show that this rate equals the rate at which the gravitational potential energy of the rod–Earth system decreases.

(e) The resistor is replaced with one of resistance $2R$, and the experiment is repeated. Is the new constant speed greater than, less than, or equal to v_T?

_____ Greater than _____ Less than _____ Equal to

Briefly justify your answer.

(f) The resistor is removed, leaving a gap between X and Y, and the rod is again released from rest. While the rod falls, is the magnitude of its acceleration greater than, less than, or equal to g?

_____ Greater than g _____ Less than g _____ Equal to g

Briefly justify your answer.

15.1. (12 points, suggested time 25 minutes) Students shine a red laser of wavelength $\lambda = 650$ nm onto a single human hair stretched vertically in front of the laser. The light forms a pattern of bright and dark fringes on a wall a distance L from the hair. The dark fringes in the pattern produced by a thin hair are at the same angles as the dark fringes in the pattern produced by a single slit whose width equals the hair's width a.

(a) The hair is replaced with a thicker hair. Does the width of the central bright fringe on the wall increase, decrease, or stay the same?

_____ Increase _____ Decrease _____ Stay the same

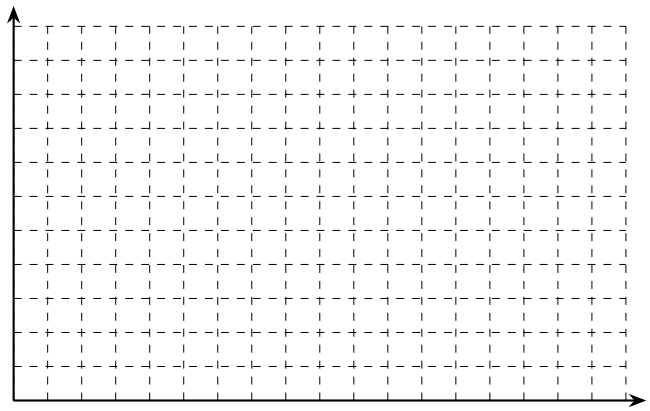
Briefly justify your answer.

(b) Derive an expression for the distance W between the two first-order dark fringes, one on each side of the central bright fringe. Assume that W is much smaller than L. Express your answer in terms of λ, L, a, and physical constants, as appropriate.

The students measure W for several distances L.

L (m)	1.0	1.5	2.0	2.5	3.0
W (mm)	16.6	24.1	32.8	39.8	49.2

(c) On the grid below, plot W as a function of L. Clearly scale and label all axes, and draw a best-fit line.



(d) Use your best-fit line to determine the width a of the hair.

(e) A student suggests measuring the distance from the center of the pattern to one dark fringe, at only one value of L, instead. Describe one way in which the students' method reduces the uncertainty in a compared with the suggested method.

(f) The red laser is replaced with a green laser of wavelength 532 nm. The same hair is used, and the wall is 2.0 m from the hair. Calculate the distance W between the two first-order dark fringes that the students should expect.

15.2. (10 points, suggested time 20 minutes) A soap film (index of refraction 1.33) is held vertically in a wire loop in air. Over time, the water drains downward, and the top of the film becomes much thinner than the wavelengths of visible light. Viewed by reflected white light, the thinnest part of the film looks black.

(a) Explain why the thinnest part of the soap film looks black in reflected light.

A glass lens (index of refraction 1.52) is given a clear coating (index of refraction 1.38) on its front surface. Consider first a coating that, like the top of the soap film, is much thinner than the wavelengths of visible light.

(b) Does this very thin coating look black in reflected light, as the thinnest part of the soap film does?

_____ Yes _____ No

Briefly explain your reasoning.

(c) The coating is instead given a thickness such that light of wavelength λ (measured in air), striking the lens at normal incidence, has as little reflection as possible. Derive an expression for the smallest coating thickness that achieves this. Express your answer in terms of λ and physical constants, as appropriate.

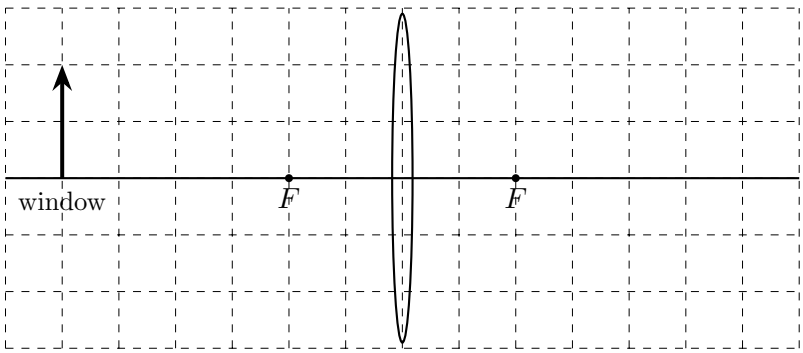
(d) The coating is made with the thickness from part (c) for $\lambda = 550$ nm (green light). When white light strikes the coated lens, the weak reflected light looks purple (a mixture of red and violet). Explain why.

Return to the soap film. Below its thinnest part, the film gradually becomes thicker.

(e) Calculate the smallest film thickness at which light of wavelength 600 nm (measured in air), striking the film at normal incidence, is reflected most strongly.

16.1. (10 points, suggested time 20 minutes) A student holds a magnifying glass, a converging lens of focal length f, between a bright window and a white wall. The window is a distance $3f$ from the lens. The student moves the wall until a sharp image of the window forms on it.

(a) In the figure below, the arrow represents the window and the points marked F are the focal points of the lens. Draw a ray diagram to locate the image of the window. Draw the image.



(b) Derive expressions for the distance from the lens to the wall and for the ratio of the image's height to the window's height. Express your answers in terms of f and physical constants, as appropriate.

(c) On your ray diagram in part (a), draw one more ray from the top of the window: a ray that strikes the lens below the axis, about halfway between the axis and the bottom edge of the lens. Continue the ray beyond the lens.

(d) The student now covers the lower half of the lens with a piece of cardboard. The lens, window, and wall do not move. What happens to the image on the wall?

_____ The top half of the image disappears

_____ The bottom half of the image disappears

_____ The whole image remains, but it is dimmer

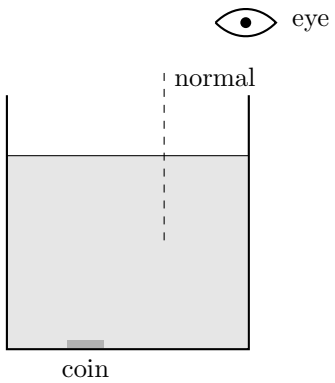
_____ The image disappears

Briefly explain your reasoning. You may refer to your ray diagram.

(e) The cardboard is removed. The student moves the lens and the wall so that a sharp image on the wall is the same size as the window. Derive expressions for the distance from the window to the lens and the distance from the lens to the wall. Express your answers in terms of f and physical constants, as appropriate.

16.2. (12 points, suggested time 25 minutes) A coin lies on the bottom of a mug of tea. The tea has depth d and index of refraction $n > 1$. A person looks down at the coin from almost directly above.

(a) In the figure below, draw a ray of light that leaves the coin and reaches the eye. Your ray should show what happens at the surface of the tea, relative to the dashed normal line. Indicate the direction in which the light travels.



A student says:

> "When the light leaves the tea, it changes speed, so it bends toward the normal. Then the coin's image is lower than the coin, so the coin looks deeper than it really is."

(b) i. Identify one aspect of the student's statement that is correct, and briefly explain why it is correct.

ii. Identify one aspect of the student's statement that is incorrect, and explain why it is incorrect.

(c) Derive an expression for the apparent depth of the coin, as seen from almost directly above. Express your answer in terms of d, n, and physical constants, as appropriate. (For small angles, $\sin\theta \approx \tan\theta$.)

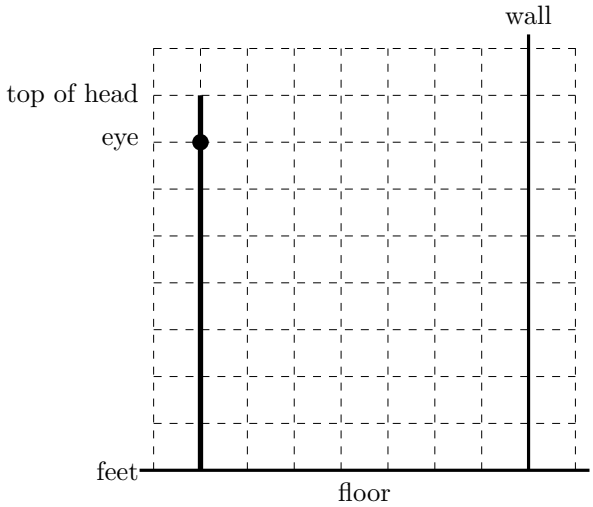
(d) The tea is replaced with the same depth of oil, which has a greater index of refraction than tea. Is the apparent depth of the coin greater than, less than, or the same as with the tea?

_____ Greater than _____ Less than _____ The same as

Briefly justify your answer.

(e) Treat the coin as a point at the bottom of the tea. Some of the light from the coin strikes the surface of the tea and does not leave the tea. Derive an expression for the radius of the circular region of the surface, centered directly above the coin, through which light from the coin can leave the tea. Express your answer in terms of d, n, and physical constants, as appropriate.

16.3. (12 points, suggested time 25 minutes) A person of height H stands a distance D in front of a vertical wall, facing it. Her eyes are a height h above the floor. She wants to hang a flat (plane) mirror on the wall so that she can see her whole body, from the top of her head to her feet, in the mirror. The figure shows the person (the vertical line) and the wall from the side.



(a) On the figure, draw a ray of light that travels from the person's feet, reflects from the wall, and reaches her eye. Also draw a ray that travels from the top of her head, reflects from the wall, and reaches her eye. Then draw a bracket along the wall to mark the shortest section of wall that the mirror must cover for her to see her whole body.

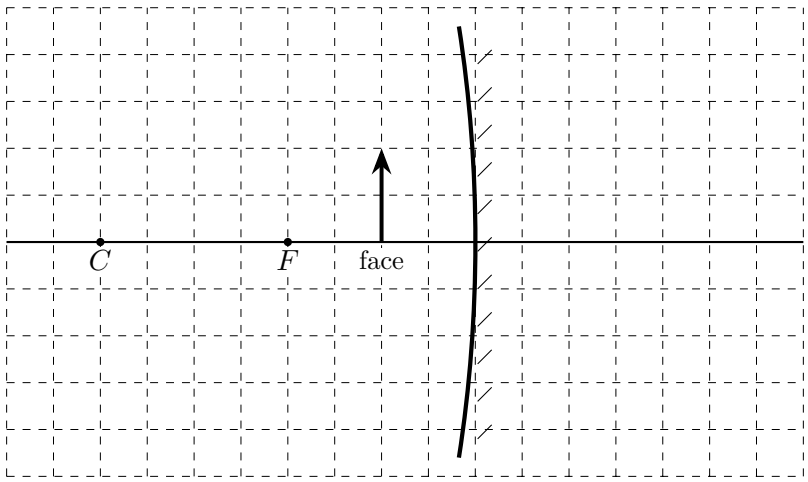
(b) Derive expressions for the smallest height the mirror can have and for the height of the mirror's bottom edge above the floor. Express your answers in terms of H, h, D, and physical constants, as appropriate.

(c) The person backs away from the wall until she is a distance $2D$ from it. Is the smallest mirror height that lets her see her whole body now greater than, less than, or the same as before?

_____ Greater than _____ Less than _____ The same as

Briefly justify your answer.

The person also uses a small concave makeup mirror of focal length f. She holds her face a distance $f/2$ in front of it. In the figure below, the arrow represents her face, and the points marked F and C are the focal point and the center of curvature of the mirror.



(d) On the figure, draw a ray diagram to locate the image of the face formed by the mirror. Draw the image.

(e) Derive an expression for the distance between the image and the mirror, and state whether the image is in front of or behind the mirror. Also derive an expression for the ratio of the image's height to the face's height. Express your answers in terms of f and physical constants, as appropriate.

(f) The person moves her face away from the makeup mirror until it is a distance $3f/4$ from the mirror. Is the height of the image now greater than, less than, or the same as the height of the image in part (e)?

_____ Greater than _____ Less than _____ The same as

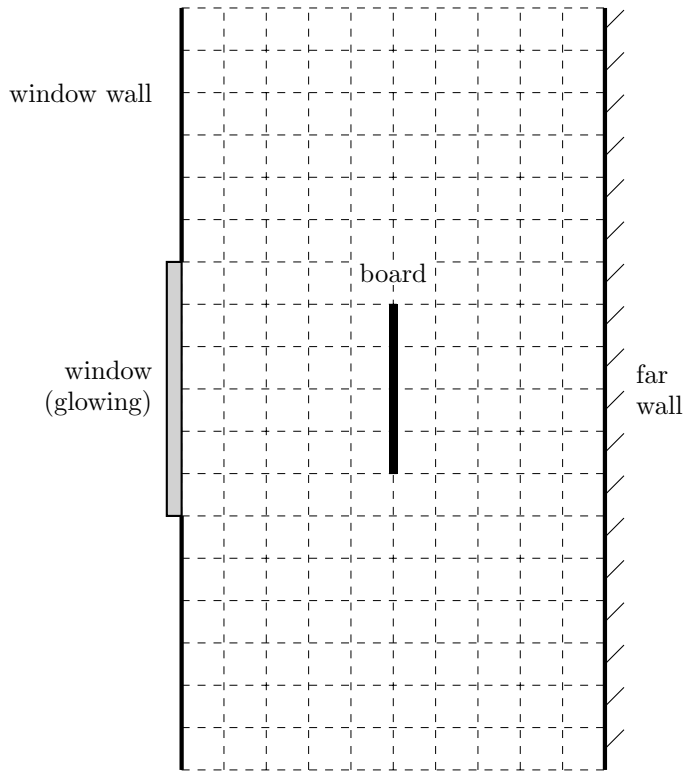
Briefly justify your answer.

(g) The concave mirror is replaced with a convex mirror whose focal length has the same magnitude, f. The face is again a distance $f/2$ from the mirror. Is the height of the image greater than, less than, or the same as the height of the face?

_____ Greater than _____ Less than _____ The same as

Briefly justify your answer.

16.4. (12 points, suggested time 25 minutes) A room has a frosted-glass window 1.20 m wide in one wall. Sunlight outside makes the frosted glass glow uniformly, so every point of the window sends light in all directions into the room. A tall, thin, opaque board 0.80 m wide stands parallel to the window wall, centered in front of the window, 1.00 m from it. The far wall of the room is parallel to the window wall and 2.00 m from it. The window, the board, and the room are much taller than they are wide, so a top view shows everything that matters. The figure shows the top view; each grid square is 0.20 m on a side. Ignore light reflected from any surface.

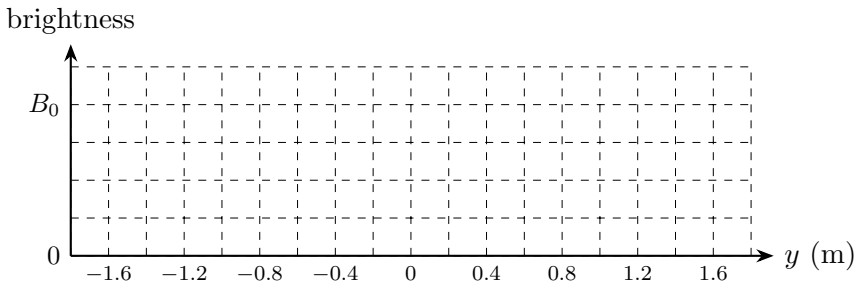


(a) On the figure, draw the rays needed to locate the boundaries of the shadow regions on the far wall. Then, along the far wall, label each region with a letter:

- F (full shadow): no light from the window reaches it.
- P (partial shadow): light from part of the window, but not all of it, reaches it.
- L (fully lit): light from every part of the window reaches it.

(b) Using geometry (not a measurement of your drawing), calculate the width of the full-shadow region on the far wall and the width of each partial-shadow region on the far wall.

(c) Model the brightness of a point on the far wall as proportional to the width of the window that can be seen from that point. On the axes below, sketch the brightness of the far wall as a function of position y along the wall, where $y = 0$ is the point directly behind the center of the board. B_0 is the brightness of a point from which the whole window can be seen.



(d) The board is moved toward the far wall until it is 1.50 m from the window wall. It stays parallel to the window wall and centered in front of the window. Is the width of each partial-shadow region on the far wall now greater than, less than, or equal to its width in part (b)?

_____ Greater than _____ Less than _____ Equal to

Briefly explain your reasoning.

(e) The board is removed. In its place, a tall, thin, flat opaque strip 0.20 m wide, parallel to the window wall, is centered in front of the window, 1.00 m from the window wall. Behind the strip is a region of the room that receives no light from the window. Calculate the greatest distance from the window wall at which a point in this region can be found. Does the far wall have a full-shadow region? Justify your answer.

(f) The strip is removed, and the board is returned to its original position, 1.00 m from the window wall. A curtain is drawn across the window, leaving uncovered only a very narrow vertical strip at the center of the window. Is the width of the region of the far wall that receives no light from the window now greater than, less than, or equal to the width of the full-shadow region in part (b)?

_____ Greater than _____ Less than _____ Equal to

Briefly explain your reasoning.

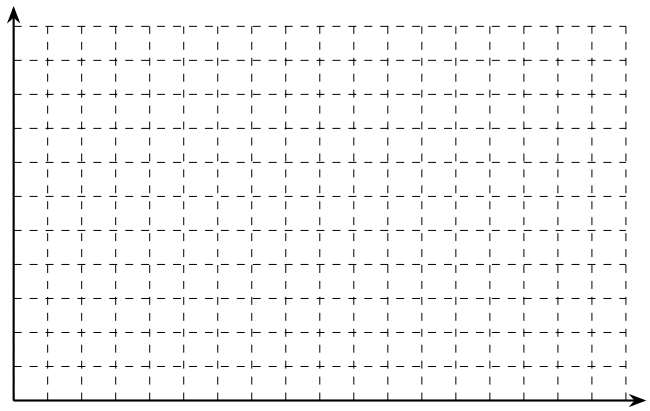
17a.1. (12 points, suggested time 25 minutes) Students connect each of five light-emitting diodes (LEDs) of different colors, one at a time, to a variable power supply. For each LED, they slowly increase the potential difference across it and record the turn-on voltage V_{on} at which it first visibly glows. The wavelength λ of each LED's light is printed on its package. Model each LED this way: when it glows, each electron that crosses the LED loses energy eV_{on} and this energy produces one photon.

(a) Using this model and energy conservation, derive an expression for V_{on} in terms of the frequency f of the LED's light and physical constants.

Color	λ (nm)	V_{on} (V)	
Red	630	1.99	
Orange	605	2.03	
Yellow	590	2.10	
Green	525	2.40	
Blue	470	2.61	

(b) Use the empty column in the table to record a quantity that, graphed against V_{on}, will produce a straight line whose slope can be used to determine Planck's constant. Label the column, including units.

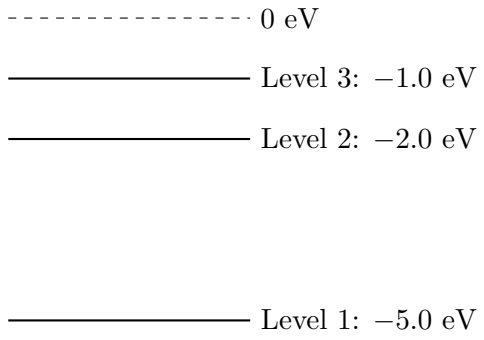
(c) On the grid below, plot V_{on} against the quantity from part (b). Clearly scale and label all axes, and draw a best-fit line.



(d) Use your best-fit line to determine a value for Planck's constant.

(e) The students also have an LED labeled 400 nm. Predict its turn-on voltage. Then state whether its light would have more or less energy per photon than the red LED's light, and briefly explain why.

17a.2. (10 points, suggested time 20 minutes) The atoms of a gas in a discharge lamp have only the three energy levels shown below. When the lamp is on, atoms are excited to both upper levels, and the lamp emits light at exactly three wavelengths, labeled by the transitions that produce them: $3 \to 1$, $2 \to 1$, and $3 \to 2$.



(a) Rank the wavelengths of the three emitted lines, from longest to shortest. If any are equal, indicate that clearly.

Longest _____ _____ _____ Shortest

Briefly justify your ranking.

(b) Calculate the wavelength of the $3 \to 1$ line.

Light from the lamp passes through a cool cloud of the same kind of atoms. In the cloud, essentially every atom is in level 1. A spectrometer on the far side of the cloud records which of the lamp's three lines are dimmed.

(c) i. Is the $2 \rightarrow 1$ line dimmed after passing through the cloud?

_____ Yes _____ No

Briefly explain your reasoning.

ii. Is the $3 \rightarrow 2$ line dimmed after passing through the cloud?

_____ Yes _____ No

Briefly explain your reasoning.

(d) The spectrometer is moved so that it views the cloud from the side, where no light comes to it directly from the lamp. Check every line that the spectrometer detects coming from the cloud.

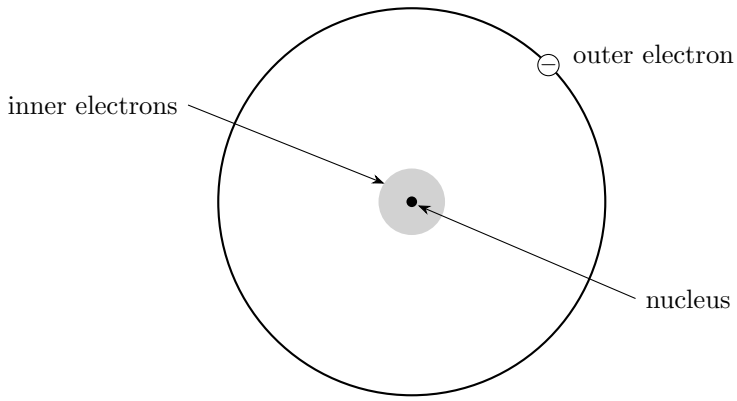
_____ $3 \to 1$ _____ $2 \to 1$ _____ $3 \to 2$

Briefly explain your reasoning.

(e) The lamp is turned off. A beam of electrons, each with kinetic energy 3.5 eV, is sent through the cool cloud, in which essentially every atom is in level 1. Determine all the possible kinetic energies of an electron just after a single collision with one atom. Justify your answer.

(f) Calculate the longest wavelength of light that can ionize an atom in the cool cloud, removing its electron completely.

17a.3. (12 points, suggested time 25 minutes) A *Rydberg atom* is an atom in which one electron has been excited to a level with a very large value of the quantum number n. Consider a rubidium atom. Its nucleus contains 37 protons. Of its 37 electrons, 36 are in inner shells close to the nucleus, and one, the outer electron, is farther out. In a Rydberg atom of rubidium, the outer electron is excited to a level with n near 50, so that it is very far outside the inner shells, as shown in the figure (not to scale).



The Bohr model of an ion with a single electron orbiting a nucleus of charge $+Ze$ gives the energy levels and orbit radii

$$E_n = -\frac{(13.6\text{ eV})Z^2}{n^2}, \qquad r_n = \frac{(0.0529\text{ nm})\,n^2}{Z}.$$

In this question, use these expressions to model the outer electron of a Rydberg atom of rubidium.

(a) What value of Z should be used to model the outer electron? Explain your reasoning.

(b) Calculate the frequency of the photon emitted when the outer electron makes a transition from $n = 51$ to $n = 50$. State the region of the electromagnetic spectrum in which this photon lies.

(c) Calculate the orbit radius of the outer electron for $n = 50$. In air at room temperature and atmospheric pressure, the average distance between neighboring molecules is about 3 nm. Explain why experiments with Rydberg atoms are done with gases at very low pressure.

(d) A student says, "The energy levels are proportional to $1/n^2$. If n is doubled, every energy is divided by 4, so the photon emitted in a transition from $n = 101$ to $n = 100$ has about $\frac{1}{4}$ the energy of the photon emitted in a transition from $n = 51$ to $n = 50$." Calculate the energy of the photon emitted in a transition from $n = 101$ to $n = 100$, and use it to evaluate the student's claim. If the claim is incorrect, explain the flaw in the student's reasoning.

(e) In the lowest-energy state of a rubidium atom, the outer electron is in the level $n = 5$. The measured energy of the outer electron in this state is -4.18 eV. In a real atom, the outer electron in a low-n state does not stay at one distance from the nucleus; it spends part of its time among the inner electrons. Calculate the energy that the model predicts for $n = 5$. Identify which assumption from your reasoning in part (a) fails for $n = 5$, and explain why the measured energy differs from the model's prediction in the way that it does.

17b.1. (10 points, suggested time 20 minutes) A polonium-210 nucleus, $^{210}_{84}\text{Po}$, at rest undergoes alpha decay. The tables below give the atomic numbers of several elements and the atomic masses of several isotopes. Use 1 u = 931.5 MeV/c^2.

Element	Atomic number Z
Mercury (Hg)	80
Thallium (Tl)	81
Lead (Pb)	82
Bismuth (Bi)	83
Polonium (Po)	84
Astatine (At)	85

Isotope	Atomic mass (u)
Helium-4	4.002603
Mercury-206	205.977514
Thallium-206	205.976110
Lead-206	205.974465
Lead-210	209.984188
Bismuth-210	209.984120
Polonium-210	209.982874

(a) Identify the nucleus produced in this decay, including its element, its atomic number, and its mass number. Explain how you determined each number.

(b) Calculate the energy released in this decay, in MeV.

After the decay, the alpha particle and the nucleus produced in the decay move apart.

(c) Is the kinetic energy of the alpha particle greater than, less than, or the same as the kinetic energy of the nucleus produced in the decay?

_____ Greater than _____ Less than _____ The same as

In a clear, coherent, paragraph-length response that incorporates physics principles, explain your reasoning.

(d) Calculate the kinetic energy of the alpha particle, in MeV. Assume that all of the energy released in the decay becomes kinetic energy of the alpha particle and the other nucleus.

(e) The polonium-210 found in nature is produced by the beta-minus decay of another nucleus. Which nucleus decays to $^{210}_{84}$Po by beta-minus decay?

_____ Lead-210 _____ Bismuth-210 _____ Astatine-210 _____ Lead-206

Briefly explain your reasoning.

17b.2. (12 points, suggested time 25 minutes) Technetium-99m (Tc-99m) is a radioactive isotope used in medical imaging. A hospital obtains it from molybdenum-99 (Mo-99), which undergoes beta-minus decay to produce Tc-99m. The atomic numbers of several elements are given below.

Element	Niobium (Nb)	Molybdenum (Mo)	Technetium (Tc)	Ruthenium (Ru)
Atomic number Z	41	42	43	44

(a) Write the nuclear equation for the beta-minus decay of ${}^{99}_{42}\text{Mo}$ that produces Tc-99m, showing the mass number and atomic number of each particle. (In the equation, write Tc-99m in the form ${}^{A}_{Z}\text{Tc}$.) Explain how conservation laws determine the mass number and atomic number of the technetium nucleus.

(b) A Tc-99m nucleus decays by emitting a single gamma-ray photon and no other particle. How do the atomic number Z and mass number A of the nucleus that remains compare with those of the Tc-99m nucleus?

_____ Same Z, same A _____ Z greater by 1, same A

_____ Same Z, A less by 1 _____ Z less by 1, same A

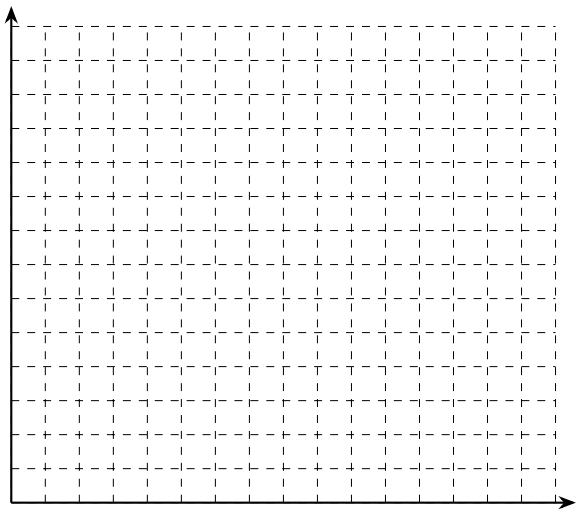
Briefly explain your reasoning.

A technician checks a vial labeled "Tc-99m" by placing it at a fixed distance from a radiation counter and recording the number of counts in one minute every 3 hours. With no sample present, the counter records a steady background count rate of 120 counts per minute, which comes from sources other than the vial. The technician's measurements are shown below. Because radioactive decay is a random process, the count rates scatter about a smooth curve.

Time t (h)	Count rate (counts/min)	
0	1070	
3	814	
6	626	
9	452	
12	366	
15	267	
18	243	
21	215	
24	179	

(c) Use the empty column in the table to record the count rate due to the sample alone. Label the column, including units.

(d) On the grid on the next page, plot the count rate due to the sample alone as a function of time. Clearly scale and label all axes, and draw a smooth best-fit curve.



(e) Use your curve to determine the half-life of the sample. Show on your graph how you obtained your value. Is your value consistent with the half-life of Tc-99m, 6.0 h?

(f) Another student graphs the count rates exactly as the technician recorded them. That student notes that the count rate falls from 1070 counts/min to half of that value in about 7.5 h, and concludes that the vial must contain some other isotope with a longer half-life. Is the student's conclusion supported by the data?

_____ Yes _____ No

Briefly explain your reasoning.

(g) A patient's scan requires a dose of Tc-99m with an activity of 740 MBq (740 million decays per second) at the time of injection. The dose is prepared 12 hours before the injection. Using a half-life of 6.0 h, calculate the activity the dose must have when it is prepared.

AP Physics 2 Chapter 18a: Ideal gases and kinetic theory

18a.1. (10 points, suggested time 20 minutes) A sealed glass bulb contains equal numbers of helium atoms and xenon atoms. A xenon atom has about 33 times the mass of a helium atom. The bulb has been sitting in a room for a long time, so the gas mixture is in thermal equilibrium at the room's temperature. Treat the mixture as an ideal gas.

(a) Is the average translational kinetic energy of a helium atom greater than, less than, or the same as that of a xenon atom?

_____ Greater than _____ Less than _____ The same as

Briefly justify your answer.

(b) Derive an expression for the ratio of the root-mean-square speed of the helium atoms to the root-mean-square speed of the xenon atoms.

(c) Consider a typical helium atom and a typical xenon atom moving at their root-mean-square speeds. Explain how the change in momentum of a helium atom when it strikes a wall compares with that of a xenon atom, and how often a helium atom strikes the walls compared with a xenon atom.

(d) Is the part of the gas pressure due to collisions of helium atoms with the walls greater than, less than, or the same as the part due to collisions of xenon atoms with the walls?

_____ Greater than _____ Less than _____ The same as

Briefly explain your reasoning in terms of the collisions of atoms with the walls.

(e) The bulb has an internal volume of 0.50 L, the room temperature is 20°C, and the total pressure of the gas mixture is 1.0×10^5 Pa. Calculate the number of helium atoms in the bulb.

(f) The bulb develops a tiny pinhole, and atoms slowly escape into the room through it. As this happens, does the ratio of the number of helium atoms to the number of xenon atoms in the bulb increase, decrease, or stay the same?

_____ Increase _____ Decrease _____ Stay the same

Briefly explain your reasoning.

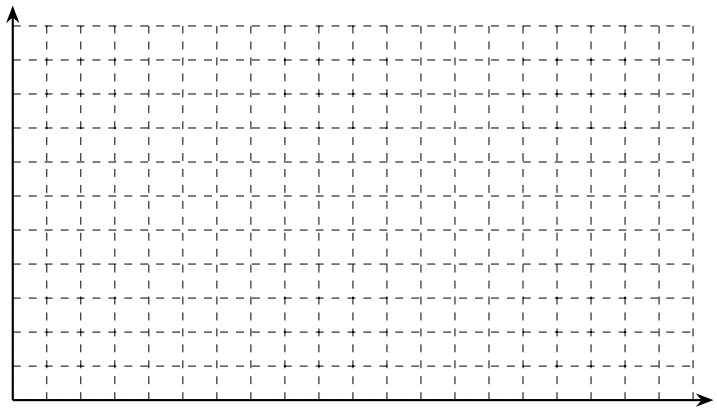
18a.2. (12 points, suggested time 25 minutes) Students trap a sample of air in a greased glass syringe by sealing its tip. The plunger slides freely without leaking, so the trapped air stays at atmospheric pressure. The students place the syringe in a series of water baths at different temperatures and read the volume of the trapped air from the syringe's scale.

(a) Describe two steps the students should take to make each volume reading reliable.

Bath temperature T (°C)	5	25	45	65	85
Volume V (mL)	20.4	21.7	23.4	24.7	26.3

(b) Using the ideal gas law, explain why the volume of the trapped air should be a linear function of its temperature in degrees Celsius.

(c) On the grid below, plot V as a function of T in degrees Celsius. Choose scales so that you can extend a best-fit line to $V = 0$. Draw the best-fit line and extend it.



(d) On your graph, mark the point where your extended line reaches $V = 0$. Use your line to estimate the temperature, in degrees Celsius, at which the volume of the trapped air would be zero. Then use the ideal gas law to explain what this temperature represents.

(e) The experiment is repeated with twice as much air trapped in a larger syringe. Would the temperature found in part (d) be greater than, less than, or the same as before?

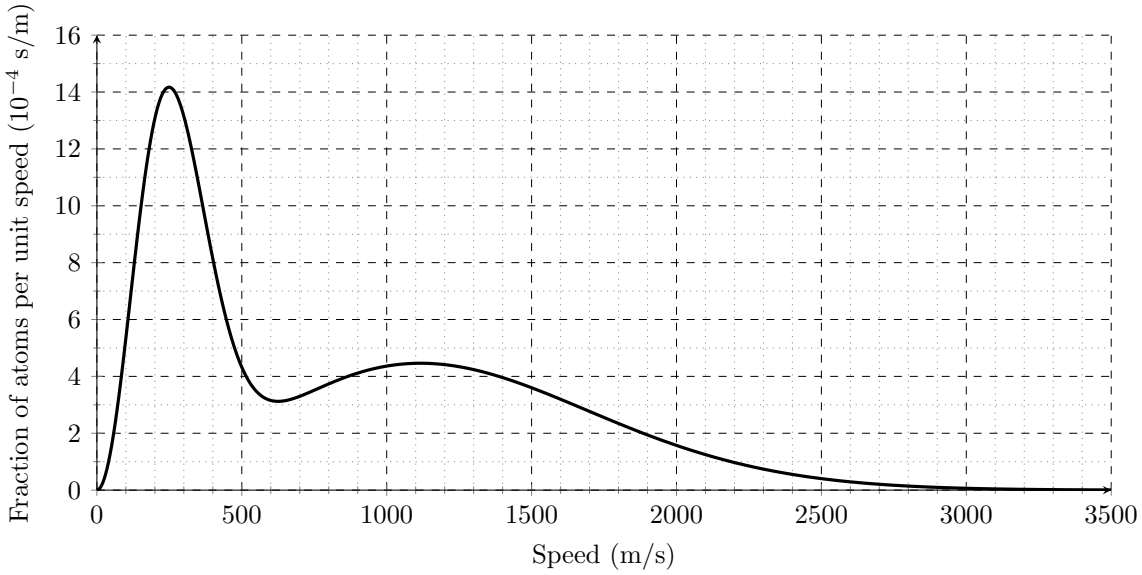
_____ Greater than _____ Less than _____ The same as

Briefly justify your answer.

(f) Atmospheric pressure is 1.0×10^5 Pa. Use the data to calculate the number of moles of air trapped in the syringe in the original experiment.

18a.3. (12 points, suggested time 25 minutes) A sealed container holds a mixture of helium atoms and krypton atoms. A krypton atom has about 21 times the mass of a helium atom. The container has been in a room for a long time, so the mixture is in thermal equilibrium at the room's temperature, 300 K. Treat the mixture as an ideal gas.

The graph shows the distribution of speeds of all the atoms in the container. For any range of speeds, the area under the curve over that range equals the fraction of all the atoms in the container that have speeds in that range. The total area under the curve is 1.



(a) Is the average translational kinetic energy of a helium atom greater than, less than, or the same as that of a krypton atom?

_____ Greater than _____ Less than _____ The same as

Briefly justify your answer.

(b) Which peak of the graph is due mostly to the helium atoms?

_____ The left peak _____ The right peak

Briefly justify your answer.

(c) For a gas of one kind of atom, the speed at the peak of the distribution is proportional to the root-mean-square speed of the atoms. Using the masses of the atoms, derive the predicted ratio of the speed at the peak due to the helium atoms to the speed at the peak due to the krypton atoms. Then use the graph, together with your answer to part (b), to determine this ratio, and state whether the two values agree.

(d) A student says, "The left peak is about three times as tall as the right peak, so the container holds more of the kind of atom that makes the left peak."

i. Explain what, if anything, is wrong with the student's reasoning.

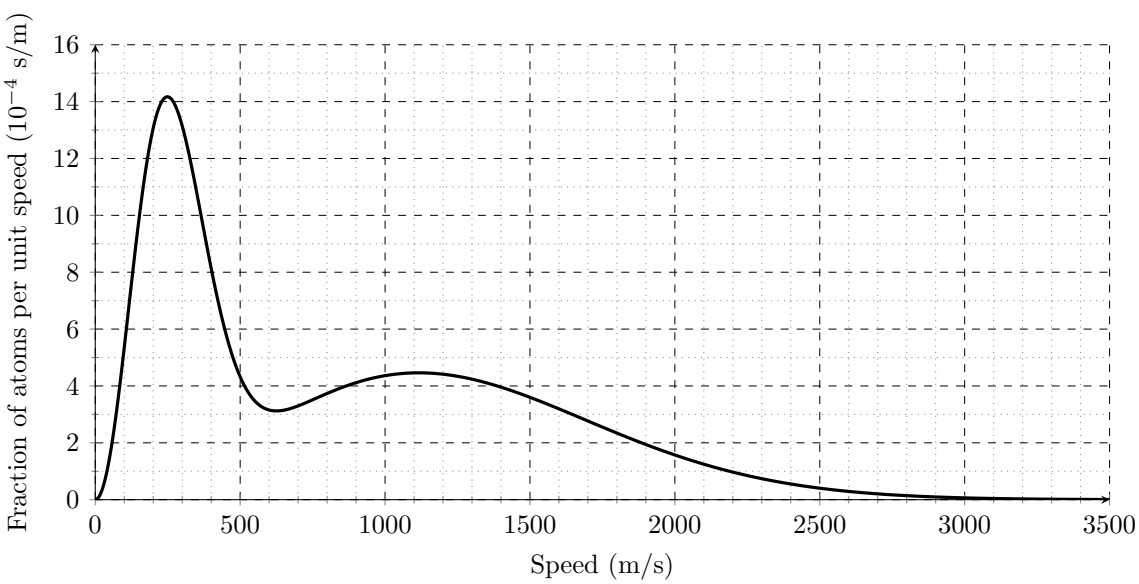
ii. Does the container hold more of the kind of atom that makes the left peak, more of the kind that makes the right peak, or equal numbers of each?

_____ More left-peak atoms _____ More right-peak atoms _____ Equal numbers

Briefly explain how the graph supports your answer.

(e) The mass of a helium atom is 6.65×10^{-27} kg. Calculate the root-mean-square speed of the helium atoms in the container. Compare your result with the speed at the peak due to the helium atoms, and explain why the two speeds differ in the way that they do.

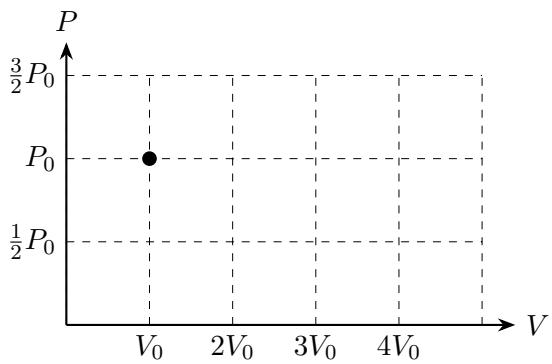
(f) The container is heated until the mixture is in equilibrium at 600 K. On the graph below, which repeats the 300 K curve, sketch the distribution of speeds at 600 K.



18b.1. (10 points, suggested time 20 minutes) A sample of monatomic ideal gas is sealed in a cylinder by a movable piston. Initially, the gas has pressure P_0, volume V_0, and temperature T_0. Two separate trials start from this same initial state.

- **Process A:** The gas is heated slowly while the pressure stays constant, until its volume is $2V_0$.
- **Process B:** The cylinder sits in a large water bath that keeps the gas at temperature T_0, and the piston is pulled out slowly until the volume is $2V_0$.

(a) On the axes below, the initial state is marked with a dot. Sketch and label process A and process B.



(b) What is the final temperature of the gas in process A? Express your answer in terms of T_0, and justify your answer.

(c) How does the work done by the gas in process A compare with the work done by the gas in process B? Explain your reasoning by referring to your graph in part (a).

(d) The temperature of the gas does not change during process B. Explain why energy must nevertheless be transferred to the gas by heating during process B.

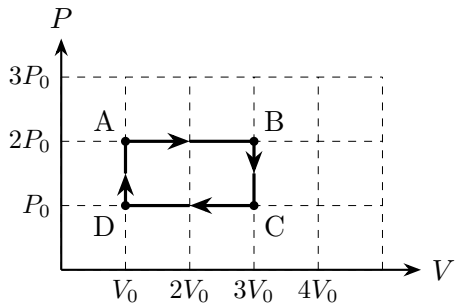
(e) In which process, if either, is more energy transferred to the gas by heating?

_____ Process A _____ Process B _____ The same in both

Briefly explain your reasoning.

(f) Derive an expression for the energy transferred to the gas by heating during process A. Express your answer in terms of P_0, V_0, and physical constants, as appropriate.

18b.3. (12 points, suggested time 25 minutes) A sample of monatomic ideal gas in a demonstration engine is taken around the cycle A→B→C→D→A shown on the pressure–volume graph below.



(a) Derive an expression for the net work done by the gas during one complete cycle. Express your answer in terms of P_0 and V_0.

A student says:

> "After one cycle the gas is back in state A, so its internal energy hasn't changed. So the net energy transferred to the gas by heating during the cycle must be zero."

(b) i. Identify one aspect of the student's statement that is correct, and briefly explain why it is correct.

ii. Identify one aspect of the student's statement that is incorrect, and explain why it is incorrect.

(c) Derive an expression for the energy transferred to the gas by heating during process A→B. Express your answer in terms of P_0 and V_0.

(d) Derive an expression for the energy transferred to the gas by heating during process B→C. Express your answer in terms of P_0 and V_0, and indicate whether this energy is transferred into or out of the gas.

(e) Rank the temperatures T_A, T_B, T_C, and T_D of the gas in the four states. If any are equal, indicate that clearly.

Highest _____ _____ _____ _____ Lowest

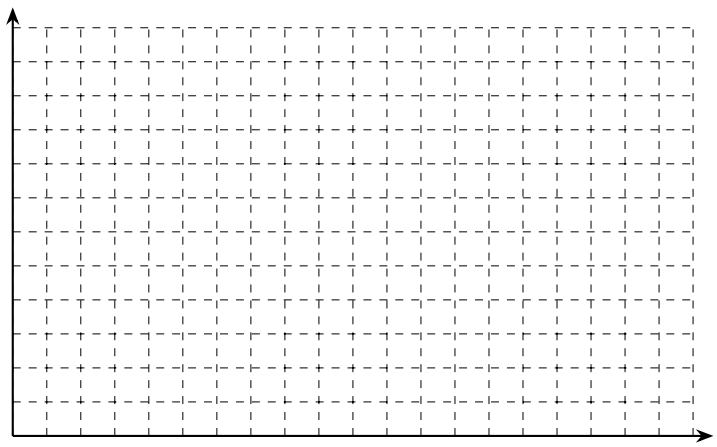
Briefly justify your ranking.

18b.4. (12 points, suggested time 25 minutes) A heat-pump water heater sits in a basement. Its refrigerant circulates around a closed loop of tubing, over and over. In a coil exposed to the basement air, the refrigerant takes in energy from the air. An electric compressor then squeezes the refrigerant. In a second coil wrapped around a water tank, the refrigerant gives off energy to the water.

The tank holds 150 kg of water, and the specific heat of water is 4190 J/(kg · °C). The tank is well insulated. An electricity meter shows that the heat pump uses electrical energy at a steady rate of 1.00 kW, and all of this energy is transferred to the refrigerant as work done by the compressor. All of the energy the refrigerant gives off in the tank coil goes into the water. Once the heat pump is running steadily, a student records the temperature of the water.

Time t (min)	0	15	30	45	60	75	90	105	120
Water temperature T (°C)	15.4	18.8	22.5	26.4	30.0	34.0	37.0	41.3	44.8

(a) On the grid below, plot the water temperature T as a function of time t. Label the axes with quantities and units, and choose scales that use most of the grid. Draw a best-fit line.



(b) Using your best-fit line, calculate the rate at which energy is transferred to the water.

(c) Calculate the rate at which the refrigerant takes in energy from the basement air. Justify your method by applying the first law of thermodynamics to the refrigerant.

A student says:

> "The heat pump transfers more energy to the water than it takes from the electric outlet. Energy can't be created, so the electricity meter must be reading too low."

(d) i. Identify one aspect of the student's statement that is correct, and briefly explain why it is correct.

ii. Identify one aspect of the student's statement that is incorrect, and explain why it is incorrect.

(e) The heat pump is replaced by an electric heating element placed in the water. The element uses electrical energy at a steady rate of 1.00 kW, all of which goes into the water. Is the time needed to warm the same tank of water from 15°C to 45°C greater than, less than, or the same as the time needed with the heat pump?

_____ Greater than _____ Less than _____ The same as

Briefly explain your reasoning.

(f) The heat pump is instead connected to a smaller insulated tank holding 100 kg of water and runs exactly as before, transferring energy to the water at the same rate. Is the slope of the temperature–time graph for the smaller tank greater than, less than, or the same as the slope of your line in part (a)?

_____ Greater than _____ Less than _____ The same as

Briefly explain your reasoning.

AP Physics 2 Chapter 18c: Entropy and the second law

18c.1. (10 points, suggested time 20 minutes) A rubber ball of mass m is held at rest a height h above a carpeted floor inside a sealed, insulated crate. The ball is released. It bounces lower and lower and finally comes to rest on the carpet. The system consists of the ball, the carpet, the air in the crate, the crate, and Earth. No energy is transferred into or out of this system.

(a) After the ball comes to rest, is the total energy of the system greater than, less than, or the same as its total energy just before the ball was released?

_____ Greater than _____ Less than _____ The same as

Briefly explain your reasoning.

(b) The total entropy of the system increases from the moment of release to the time the ball comes to rest. Explain why, by describing what happens to the energy that was at first associated with the ball's height. Do not use counts of microstates.

A student says:

> "The reverse could happen too. The carpet and the air could cool down a tiny bit and give their energy back to the ball, which would then jump up to height h. That would conserve energy, so physics allows it. We just never wait long enough to see it."

(c) i. Identify one aspect of the student's statement that is correct, and briefly explain why it is correct.

ii. Identify one aspect of the student's statement that is incorrect, and explain why it is incorrect.

(d) Measurements show that a fraction f of the energy converted by the time the ball comes to rest ends up in the ball itself, warming it uniformly. The specific heat of the rubber is c. Derive an expression for the increase in the ball's temperature. Express your answer in terms of f, h, c, and physical constants, as appropriate.

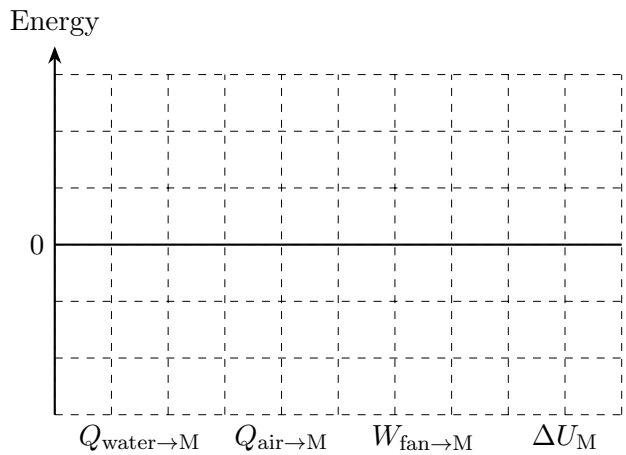
(e) Later, the insulation is removed from the sealed crate, which sits in a cooler room. The ball, carpet, air, and crate cool back to their original temperature. During this cooling, does the entropy of the ball, carpet, air, and crate increase, decrease, or stay the same?

_____ Increase _____ Decrease _____ Stay the same

Briefly explain your reasoning.

18c.2. (12 points, suggested time 25 minutes) A thermoelectric module (M) is a thin plate that produces electrical energy when energy passes through it from one face to the other. One face of the module is pressed against the bottom of a mug containing a mass m of hot water, and the other face carries metal fins exposed to the room air. Wires connect the module to a small fan. The mug has an insulated lid and sides, so energy leaves the water only through the module.
Over a time interval Δt, the water cools from temperature T_1 to temperature T_2, and the module delivers electrical energy to the fan at an average rate P. The specific heat of water is c. Assume that the internal energy of the module does not change during the interval.

(a) On the axes below, draw a bar for each quantity to represent the energy transferred to the module during the interval and the change in the module's internal energy. A bar below the zero line means energy left the module. Relative bar lengths should be consistent with energy conservation.



(b) Derive an expression for the energy transferred from the module to the room air during the interval. Express your answer in terms of m, c, T_1, T_2, P, and Δt.

A student proposes:

> "The energy that goes out through the fins is wasted. Wrap the fins in insulation so that no energy goes into the room air. Then all of the energy that leaves the water will become electrical energy for the fan."

(c) i. Identify one aspect of the student's proposal that is correct, and briefly explain why it is correct.

ii. Identify one aspect of the student's proposal that is incorrect, and explain why it is incorrect.

(d) The original setup, with uninsulated fins, is left running. When the water has cooled to room temperature, the fan stops. Another student says, "The fan stops because the water has run out of internal energy." Explain what is wrong with this statement, and explain why the fan stops.

(e) During the interval Δt in the original setup, does the entropy of the water increase, decrease, or stay the same?

_____ Increase _____ Decrease _____ Stay the same

Briefly explain your reasoning.

(f) Consider the system of the water, the module, the fan, and the air in a closed room, with no energy entering or leaving the room. During the interval Δt, is the magnitude of the entropy change of everything other than the water greater than, less than, or equal to the magnitude of the entropy change of the water?

_____ Greater than _____ Less than _____ Equal to

Briefly explain your reasoning.

AP Physics 2 Chapter 18d: Supplement: microstates (beyond AP Physics 2)

18d.1. (10 points, suggested time 20 minutes) *This question goes beyond the AP Physics 2 course: parts (e) and (f) calculate entropy changes, which the course treats only qualitatively.*
Two identical rigid canisters each contain the same amount of the same gas. Canister X is brought to a higher temperature T_H, and canister Y is kept at a lower temperature T_C. The canisters are insulated from the rest of the world, except that when they are brought together, they are in thermal contact with each other.

(a) When the canisters are brought into contact, does thermal energy, on average, flow from canister X to canister Y, from canister Y to canister X, or neither way?

_____ From X to Y _____ From Y to X _____ Neither way

Briefly explain your reasoning in terms of entropy. (Do not rely on the statement that heat flows from a hotter object to a colder one; that is what is to be explained.)

(b) A long time after contact is established, is the entropy of canister X greater than, less than, or equal to its entropy just before contact?

_____ Greater than _____ Less than _____ Equal to

Briefly explain your reasoning.

(c) Once the canisters reach a common temperature, thermal energy no longer flows, on average, in either direction. Explain why, in terms of entropy.

For parts (d)–(f), each canister contains 0.20 mol of a monatomic ideal gas, $T_H = 400$ K, and $T_C = 300$ K. The canisters themselves absorb a negligible amount of energy.

(d) Calculate the energy transferred from one canister to the other between the moment of contact and the time the canisters reach a common temperature.

(e) When a canister's temperature changes by only a small fraction of its value, the canister's change in entropy is approximately $\Delta S \approx Q/T_{\text{avg}}$, where Q is the energy transferred to the canister by heating and T_{avg} is the average of its initial and final temperatures. Use this to calculate the change in entropy of each canister and the change in the total entropy of the two canisters.

(f) The experiment is repeated with $T_H = 500$ K and $T_C = 400$ K, so that the temperature difference is again 100 K. Is the change in the total entropy of the two canisters greater than, less than, or equal to the change in part (e)?

_____ Greater than _____ Less than _____ Equal to

Briefly explain your reasoning.

18d.2. (10 points, suggested time 20 minutes) *This question goes beyond the AP Physics 2 course: it uses microstates and $S = k_B \ln W$.*

In a simple model of a solid, each atom is an oscillator that can hold any whole number of identical energy quanta. A microstate is one particular way of assigning the quanta to the oscillators: for example, with two oscillators sharing one quantum, "first oscillator 1, second 0" and "first 0, second 1" are two different microstates. All accessible microstates of an isolated system are equally likely, and the entropy of a system with W accessible microstates is $S = k_B \ln W$.

Two tiny model solids, A and B, each consist of 3 oscillators. The table gives the number of microstates W for 3 oscillators sharing q quanta.

Number of quanta q	0	1	2	3	4	5
Number of microstates W	1	3	?	10	15	21

(a) By listing them, show how many microstates are available when 3 oscillators share 2 quanta.

(b) At first, solid A holds 4 quanta and solid B holds none, and the two solids cannot exchange energy. Calculate the total entropy of the two solids, in terms of k_B.

The solids are then placed in thermal contact, so quanta can pass between them, and the pair is isolated from everything else. The pair still has 4 quanta in all.

(c) Complete the table below. q_A and q_B are the numbers of quanta in A and B, and W_A and W_B are the numbers of microstates of each solid.

q_A	4	3	2	1	0
q_B	0	1	2	3	4
W_A					
W_B					
$W_A \times W_B$					

(d) A long time after contact, which split of the quanta is the pair most likely to be found in?

$q_A =$ _____ $q_B =$ _____

Briefly explain your reasoning.

(e) Calculate the change in the total entropy of the pair from before contact to a long time after contact. Express your answer in terms of k_B.

(f) Temperature can be defined by $\frac{1}{T} = \frac{\Delta S}{\Delta U}$ for a small change in a solid's energy at fixed size. Before contact, which solid had the higher temperature?

_____ Solid A _____ Solid B _____ Neither: the temperatures were equal

Briefly explain your reasoning using the table of microstates.

(g) A student watches the pair (with a perfect microscope) for a long time after contact and says: "Every so often, solid A holds all 4 quanta again. Each time that happens, the entropy of an isolated system has decreased, so the second law of thermodynamics is wrong." Explain what is wrong with the student's reasoning.

AP Physics 1: Scoring Guidelines

Prepared with assistance from Claude, an AI model made by Anthropic. "Example response" entries show one acceptable response, not a required one. **Trap** notes tell the tutor what a careless or pattern-matching reader is likely to write. Numerical answers in these guidelines use $g = 9.8\ \mathrm{m/s^2}$; answers that use $g = 10\ \mathrm{m/s^2}$ earn full credit unless the question specifies 9.8.

Chapter 1: One-dimensional kinematics

Question 1.1 (Two-phase toy car, page 6) — 12 points

(a)(i) — **1 point**

For the estimate $v_1\,\Delta t_{\mathrm{TOT}}$ with the correct reasoning that the car moves at v_1 for nearly all of Δt_{TOT}, and displacement is velocity times duration (no credit for this part without the correct estimate) — **1 point**

Example response: $v_1 \Delta t_{\mathrm{TOT}}$. The first phase takes up nearly all of the total time, so the car moves at v_1 almost the whole time, and its displacement is that velocity times the total time.

(a)(ii) — **1 point**

For the estimate $\dfrac{v_1 + v_2}{2}\,\Delta t_{\mathrm{TOT}}$ with the correct reasoning that nearly all of Δt_{TOT} is spent speeding up uniformly from v_1 to v_2, and for constant acceleration the average velocity is the mean of the initial and final velocities, $\frac{v_1+v_2}{2}$ (no credit for this part without the correct estimate) — **1 point**

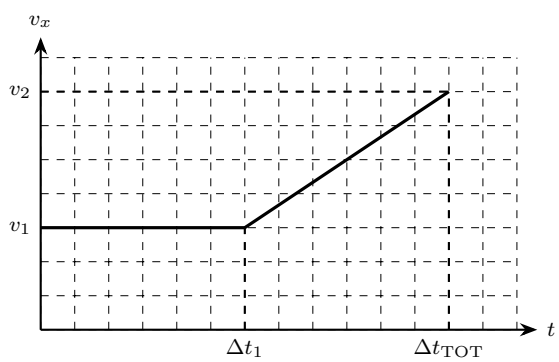
Example response: $\frac{v_1+v_2}{2}\Delta t_{\mathrm{TOT}}$. Almost all of the time is spent speeding up steadily from v_1 to v_2. With constant acceleration the average velocity is halfway between, so the displacement is that average times the total time.

Trap: Writing $v_2 \Delta t_{\mathrm{TOT}}$ (the final velocity) or $\frac{1}{2} a\,\Delta t^2$ (which assumes a start from rest).

(b) — **2 points**

For a horizontal segment at height v_1 from $t = 0$ to $t = \Delta t_1$, with endpoint values labeled — **1 point**

For a straight sloped segment from $(\Delta t_1, v_1)$ to $(\Delta t_{\mathrm{TOT}}, v_2)$ (or $\Delta t_1 + \Delta t_2$), with endpoint values labeled — **1 point**



Courtesy figure. Points attach to the features described in words.

(c) — **3 points**

For using a kinematics relationship (an equation or the area under the graph) to get the first-phase displacement $v_1 \Delta t_1$ — **1 point**

For using a kinematics relationship to get the second-phase displacement $\frac{v_1+v_2}{2}\Delta t_2$ — **1 point**

For combining them and eliminating $\Delta t_2 = \Delta t_{\mathrm{TOT}} - \Delta t_1$: $\Delta x = v_1 \Delta t_1 + \dfrac{v_1 + v_2}{2}\,(\Delta t_{\mathrm{TOT}} - \Delta t_1)$ — **1 point**

Equivalent forms earn full credit, for example $\frac{v_1+v_2}{2}\Delta t_{TOT} - \frac{v_2-v_1}{2}\Delta t_1$. *A second-phase displacement written as* $v_1 \Delta t_2 + \frac{1}{2} a \Delta t_2^2$ *earns the second point only if* a *is then replaced with* $\frac{v_2-v_1}{\Delta t_2}$, *since* a *is not one of the allowed quantities.*

(d) — **1 point**

For selecting "Yes" and correctly explaining from the equation that as $\Delta t_1 \to 0$, both the term $v_1 \Delta t_1$ and the $-\Delta t_1$ inside the parentheses go to zero, leaving $\frac{v_1+v_2}{2}\Delta t_{\text{TOT}}$, which matches (a)(ii) (no credit for this part without the correct selection) **1 point**

The correct selection is judged against the student's own work: "No" earns the point if the student's equation in (c) really does disagree with the student's answer in (a)(ii) and the explanation shows why.

(e) **1 point**

For selecting "$a_{\text{I}} < a_{\text{II}}$" and correctly explaining that the displacement is the area under the velocity–time graph and must be the same in both trials, with both graphs ending at v_2 at Δt_{TOT}; keeping the same acceleration with a higher v_1 would add area, so to restore the area the sloped segment through that fixed end point must start later and be steeper, which is a larger acceleration (no credit for this part without the correct selection) **1 point**

An algebraic explanation also earns the point if it shows that the acceleration needed grows as v_1 grows (see (f)). Reasoning only that "the car reaches v_2 in a shorter time" is incomplete, because the change in velocity $v_2 - v_1$ is also smaller; it does not earn the point.

Example response: The displacement is the area under the velocity–time graph, and it has to be the same in both trials. Both graphs end at the same point, v_2 at Δt_{TOT}. If the second trial kept the same acceleration, its sloped segment would lie on the same line as before, and the graph would differ only by the higher flat segment at the start, so the area would be larger. To bring the area back down while still ending at the same point, the car has to keep its starting speed longer and then speed up along a line that starts later and is steeper. A steeper line on a velocity–time graph is a larger acceleration, so $a_{\text{I}} < a_{\text{II}}$.

Trap: "$a_{\text{I}} > a_{\text{II}}$: the car starts closer to v_2, so it needs less acceleration." That would be true for a fixed second-phase duration, but the duration is not fixed here; the fixed displacement controls it.

(f) **3 points**

For a displacement relationship that includes the second-phase duration Δt_2 and eliminates $\Delta t_1 = \Delta t_{\text{TOT}} - \Delta t_2$, for example $\Delta x = v_1(\Delta t_{\text{TOT}} - \Delta t_2) + \frac{v_1+v_2}{2}\Delta t_2 = v_1 \Delta t_{\text{TOT}} + \frac{1}{2}(v_2 - v_1)\Delta t_2$ (from the equation in (c) or the area under the graph in (b)) **1 point**

For using $a = \dfrac{v_2 - v_1}{\Delta t_2}$ and eliminating Δt_2, for example with $\Delta t_2 = \dfrac{2(\Delta x - v_1 \Delta t_{\text{TOT}})}{v_2 - v_1}$ **1 point**

For $a = \dfrac{(v_2 - v_1)^2}{2(\Delta x - v_1 \Delta t_{\text{TOT}})}$ **1 point**

Equivalent forms earn full credit. A route through $v_2^2 = v_1^2 + 2a\,\Delta x_2$ with $\Delta x_2 = \Delta x - v_1 \Delta t_1$ also works if Δt_1 is then eliminated.

Question 1.2 (Braking car, page 9) **7 points**

(a) **3 points**

For reasoning that the car is moving more slowly throughout the second interval than throughout the first: every speed in the second interval is below $v/2$, and every speed in the first is above it **1 point**

For reasoning that the two intervals last equally long, because with constant acceleration the speed drops at a constant rate and drops by $v/2$ in each interval **1 point**

For a logical, relevant paragraph that combines these ideas (a slower car moving for the same time covers less distance) to conclude that the displacement is less in the second interval, without contradictory or incorrect statements **1 point**

Example response: The acceleration is constant, so the car loses speed at a steady rate. It loses $v/2$ of speed in the first interval and another $v/2$ in the second, so the two intervals take the same time. During the first interval the car is always moving faster than $v/2$, and during the second it is always moving slower than $v/2$. Moving more slowly for the same amount of time, it covers less distance in the second interval.

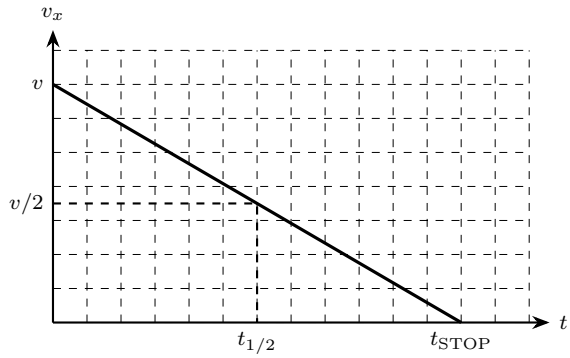
The answer blanks are not scored. An explanation that compares speeds but not durations earns only the first point: a slower car can cover more distance if it travels for longer. An energy route earns the first two points: the braking force is constant, so the distance traveled in each interval is proportional to the kinetic energy lost in it; kinetic energy depends on the square of the speed, so at $v/2$ the car has $\frac{1}{4}$ of its original kinetic energy, and it loses $\frac{3}{4}$ of it in the first interval and only $\frac{1}{4}$ in the second.

Trap: "The same: half the speed is lost in each half." Losing equal amounts of speed takes equal times, not equal distances.

(b) **2 points**

For a straight line with negative slope from v at $t = 0$ to zero at t_{STOP} **1 point**

For labels showing $v/2$ reached at $t_{1/2}$ located halfway between $t = 0$ and t_{STOP} **1 point**



Courtesy figure. Points attach to the features described in words.

(c) **2 points**

For correct expressions for both displacements by any valid kinematics. Examples: with $t_{1/2} = \frac{1}{2}t_{\text{STOP}}$ established (from $v_x = v + a_x t$ with constant a_x, or from the straight-line graph), $\Delta x_1 = \frac{v+v/2}{2}\,t_{1/2} = \frac{3}{4}v\,t_{1/2}$ and $\Delta x_2 = \frac{v/2+0}{2}\,t_{1/2} = \frac{1}{4}v\,t_{1/2}$; the trapezoid and triangle areas under the graph in (b); $\Delta x = v_0 t + \frac{1}{2}a_x t^2$ for each interval; or $v_x^2 = v_{x0}^2 + 2a_x\Delta x$ for each interval, giving $\Delta x_1 = \frac{(v/2)^2 - v^2}{2a_x} = -\frac{3v^2}{8a_x}$ and $\Delta x_2 = \frac{0-(v/2)^2}{2a_x} = -\frac{v^2}{8a_x}$ (with $a_x < 0$) **1 point**

For the ratio $\dfrac{\Delta x_2}{\Delta x_1} = \dfrac{1}{3}$ **1 point**

No particular method is required. A ratio that is inverted (3) earns 1 of 2.

Trap: Answering $\frac{1}{4}$: that is the second displacement as a fraction of the *total* stopping distance, not of the first displacement.

Question 1.3 (Race to the finish line, page 10) **7 points**

(a) **3 points**

Correct selection: "Greater than." No point is awarded for the selection itself.

For reasoning that average x-velocity is x-displacement divided by the time interval, and that the interval is the same for both cars because the race ends, for both, at the instant Car A crosses the finish line **1 point**

For reasoning that at that instant Car B has not yet reached the finish line, so its displacement is smaller than Car A's, and concluding from the same time interval that Car A's average x-velocity is greater **1 point**

For a response that is logical, follows from start to finish, and contains no incorrect or irrelevant statements **1 point**

Example response: Average x-velocity is displacement divided by elapsed time. The race lasts until Car A crosses the finish line, so the elapsed time is the same for both cars. At that moment Car B hasn't reached the finish line yet, so its displacement is smaller, and so is its average x-velocity.

Trap: "The same, because both cars cover the whole track," or reasoning from Car B's later finishing time. The race (and the interval) ends when Car A finishes; Car B's own finish is not part of it.

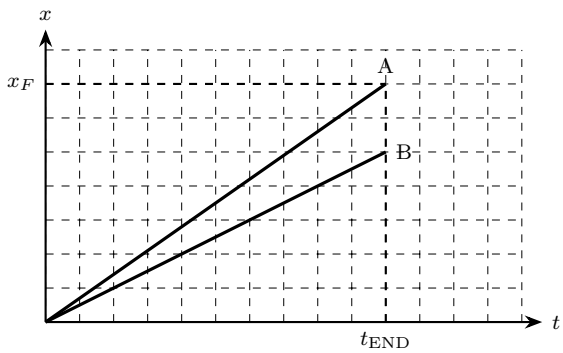
(b) **4 points**

For two graphs starting at $x = 0$, $t = 0$ that become straight lines almost at once (constant velocity) **1 point**

For Car A's line steeper than Car B's **1 point**

For both graphs ending at the same time, labeled t_{END} **1 point**

For Car A's graph ending at the labeled position x_F, with Car B's ending below it **1 point**



Courtesy figure. Points attach to the features described in words.

Chapter 2: Vectors and two-dimensional kinematics

Question 2.1 (Cannonball from a cliff, page 11) **7 points**

(a) **6 points**

Correct answer: "Less than doubled." The points describe one line of reasoning; an algebraic route through $t = \dfrac{v_{y0} + \sqrt{v_{y0}^2 + 2gH}}{g}$ *that explains why doubling* v_{y0} *less than doubles* t *(for example,* $\sqrt{4v_{y0}^2 + 2gH} < 2\sqrt{v_{y0}^2 + 2gH}$*) earns the corresponding points. A route through the greatest height also earns the second through fifth points if each step is explained: the rise time doubles; the greatest height above the launch point,* h'*, quadruples, so the greatest height above the ocean goes from* $H + h'$ *to* $H + 4h'$*, which is less than four times as great; the fall from rest through that height takes a time proportional to the square root of the height, so the fall time less than doubles; a doubled rise time plus a less-than-doubled fall time is less than double the total. No particular method is required. The answer blanks are not scored.*

For reasoning that the time of flight is set by the vertical motion alone, and that with the direction unchanged, doubling the launch speed doubles the vertical component of the launch velocity **1 point**

For reasoning that the time to rise and come back down to the launch height is proportional to the vertical launch velocity component, so that part of the flight exactly doubles **1 point**

For reasoning that when the cannonball comes back down past the launch height, it is moving downward with the same vertical speed it was launched with, which is now twice as large **1 point**

For reasoning that the rest of the fall, through the height of the cliff, starts with a greater downward speed and covers the same height, so it takes less time than before (not more) **1 point**

For combining: the total is a doubled part plus a part that got shorter, so the total is less than doubled **1 point**

For a response that is logical, follows from start to finish, and contains no incorrect or irrelevant statements **1 point**

Example response: Only the vertical motion decides when the ball splashes down. Keeping the direction and doubling the speed doubles the upward part of the launch velocity. Going up and coming back to the launch height takes a time proportional to that upward velocity, so this part of the trip doubles, and the ball passes the launch height moving down at the doubled speed. It then still has to fall the height of the cliff, but now it starts that fall faster, so that last part takes less time than it did before. A doubled part plus a shortened part adds up to less than double the original time of flight.

Trap: "Exactly doubled, because time is proportional to speed." That is true only for the part of the flight that ends at launch height (or for a launch from ground level). The cliff adds a part of the fall that does not scale with launch speed.

(b) **1 point**

For selecting "More than doubled" and correctly explaining that the vertical component of the launch velocity doubles, so the ball rises for twice as long (the time to stop rising is proportional to that component) with twice the average vertical speed, and the height above the launch point is four times as great; or, equivalently, that $h = \dfrac{v_{y0}^2}{2g}$ quadruples when v_{y0} doubles (no credit for this part without the correct selection) **1 point**

Trap: "Exactly doubled, because the launch speed doubled." The height depends on the square of the vertical launch velocity component.

Question 2.2 (Seed pod from a climbing drone, page 12) **7 points**

(a) **1 point**

For selecting "Greater than" and correctly explaining that at release the pod has the drone's velocity, including the upward component u, so it first rises before falling; it is therefore in the air longer than an object starting from rest at height H, and with the same horizontal speed w it travels farther (no credit for this part without the correct selection) **1 point**

Example response: When the pod is released it is still moving up at u, like the drone. It rises a little before it falls, so it stays in the air longer than $\sqrt{2H/g}$. Its horizontal speed is still w, so it goes farther.

Trap: "Equal to: a released object falls from rest." The pod starts with the drone's velocity, not from rest.

(b) **3 points**

For a vertical-motion equation that includes the upward initial velocity: $0 = H + ut - \frac{1}{2}gt^2$ (or an equivalent split into rising and falling stages) **1 point**

For solving for the time of flight, $t = \dfrac{u + \sqrt{u^2 + 2gH}}{g}$ (taking the positive root) **1 point**

For $D = wt = \dfrac{w\left(u + \sqrt{u^2 + 2gH}\right)}{g}$ **1 point**

A two-stage method (rise to the top in u/g, then fall from height $H + \frac{u^2}{2g}$) gives the same result and earns full credit. No particular method is required.

(c) **2 points**

For the condition: the drone is not rising at release ($u = 0$, moving only horizontally); also accept the approximate condition $u \ll \sqrt{2gH}$ with a correct explanation **1 point**

For substituting $u = 0$ into the expression from (b) to get $\dfrac{w\sqrt{2gH}}{g} = w\sqrt{\dfrac{2H}{g}}$ **1 point**

Score (c) for consistency with the expression the student derived in (b), as long as that expression contains u.

(d) **1 point**

For selecting "Less than" and correctly explaining that the pod starts with the drone's downward vertical velocity, so it reaches the ground sooner than an object starting from rest at height H, and with the same horizontal speed w it travels a shorter distance; or that replacing u with $-u$ in the expression from (b) gives $D = \dfrac{w\left(\sqrt{u^2 + 2gH} - u\right)}{g}$, which is less than $w\sqrt{\dfrac{2H}{g}}$ (no credit for this part without the correct selection) **1 point**

Trap: "Equal to: the horizontal motion does not depend on the vertical motion." The horizontal velocity is unchanged, but the time in the air is shorter.

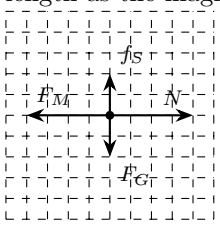
Chapter 3: Forces and Newton's laws

Question 3.1 (Fridge magnet, page 13) **7 points**

(a) **2 points**

For drawing four forces and only four forces, each labeled distinctly: downward gravitational force, upward static friction force, horizontal magnetic force toward the door, horizontal normal force away from the door **1 point**

For drawing the friction arrow the same length as the weight arrow and the normal arrow the same length as the magnetic arrow **1 point**



door on the left

Courtesy figure. The points attach to the features described in words. The horizontal pair need not be longer than the vertical pair for the second point, although the stem says $F_M > mg$. A response with an extra force (for example a "holding force" or a "net force") earns 0 for the first point.

(b) **1 point**

For selecting "The same as" and correctly explaining that the magnet's vertical acceleration is zero, so by Newton's second law the upward static friction force balances the downward gravitational force, which has not changed; the increase in the normal force does not change the friction force actually exerted (no credit for this part without the correct selection) **1 point**

Example response: The magnet stays at rest, so the vertical forces on it add to zero. The only vertical forces are the downward weight and the upward static friction. The weight did not change, so the friction force is still mg. Pressing harder raises the maximum possible friction, $\mu_S N$, but not the friction actually exerted.
Trap: "Greater than, because $f = \mu N$ and N increased." The formula gives the largest possible static friction, not the friction exerted. The question asks for the friction force, not its maximum. Earns 0.

(c) **1 point**

For selecting "Greater than" and correctly explaining that the magnet's horizontal acceleration is zero, so the normal force now balances the magnetic force plus the finger's force and is larger than F_M; the greatest possible static friction force is $\mu_S N$, so it is larger (no credit for this part without the correct selection) **1 point**

Example response: Horizontally the magnet is at rest, so the door's normal force balances both the magnetic force and the push of the finger: $N = F_M + F_{\text{finger}}$, which is more than before. The greatest static friction the door could exert is $\mu_S N$, so it increases, even though the friction actually exerted stays mg.
An explanation that says only "pressing harder increases friction," without the horizontal force balance that makes the normal force larger, does not earn the point.

(d) **3 points**

For applying the vertical force balance with the static friction force on the magnet at its maximum value, $\mu_S N$, at the point of slipping **1 point**

For using $N = F_M$ (from the horizontal force balance), not $N = mg$ **1 point**

For including the weights of both the magnet and the calendar, $\mu_S F_M = (m + m_c)g$ (or the equivalent pair of balances on the magnet and on the calendar separately), and the correct expression $m_c = \dfrac{\mu_S F_M}{g} - m$ **1 point**

Trap: Two misreadings. (1) Using $N = mg$ out of habit from tabletop problems gives $m_c = (\mu_S - 1)m$, which is negative unless $\mu_S > 1$; a student who reports it has not checked the result. Earns 1 of 3. (2) Forgetting that the magnet's own weight also needs friction gives $\mu_S F_M / g$; earns 2 of 3.

Question 3.2 (Block and hanging mass, page 14) — 12 points

(a)(i) **2 points**

For identifying as correct that doubling the hanging mass doubles the net force on the system along the direction of motion, because that net force is the weight Mg of the hanging mass (the block's weight and the normal force cancel, and the string tension is internal to the system) **1 point**

For identifying as correct the use of Newton's second law and the qualitative conclusion that the acceleration increases, because the net force doubles while the total mass increases by a smaller factor **1 point**

(a)(ii) **2 points**

For identifying as incorrect the claim that the system's inertial mass stays the same: the hanging mass is accelerated too, so the system's inertial mass is $M + M = 2M$ before and $M + 2M = 3M$ after **1 point**

For identifying as incorrect the conclusion $a_2 = 2a_1$: because the inertial mass also increases, the acceleration increases by a factor less than 2 **1 point**

(b) **4 points**

For applying Newton's second law to the block–string–mass system: $a = \dfrac{\Sigma F}{m_{\text{sys}}}$ **1 point**

For the net force along the direction of motion: Mg in the first experiment and $2Mg$ in the second **1 point**

For the system mass: $2M$ in the first experiment and $3M$ in the second **1 point**

For the correct result $a_2 = \frac{4}{3}a_1$, from $a_1 = \frac{Mg}{2M} = \frac{g}{2}$ and $a_2 = \frac{2Mg}{3M} = \frac{2g}{3}$ **1 point**

Separate free-body diagrams for the block and the hanging mass, with the tension eliminated, earn all four points if they reach the same result.

(c)(i) **2 points**

For pointing to the numerator (Mg becoming $2Mg$) as the expression of the correct claim that the net force doubles **1 point**

For pointing to $a_2 > a_1$ (from $\frac{2}{3} > \frac{1}{2}$) as the expression of the correct claim that the acceleration increases **1 point**

(c)(ii) **2 points**

For pointing to the denominator ($2M$ becoming $3M$) as the correction of the claim that the inertial mass is unchanged **1 point**

For explaining that because the denominator grows by $\frac{3}{2}$ while the numerator doubles, the acceleration grows by $\frac{2}{3/2} = \frac{4}{3}$, not 2 **1 point**

Trap: The student's argument is mostly right, so a reader who skims it agrees with it. The error is the single sentence about which part of the system has inertia.

Question 3.3 (Block pushed by a panel, page 16) 7 points

(a) **6 points**

Correct selection: "Greater than half." No point is awarded for the selection itself.

For identifying the forces on the block: the downward gravitational force and the downward normal force from the panel (and no others) **1 point**

For applying Newton's second law with both forces: the acceleration comes from the sum of the two downward forces divided by the mass, $ma = mg + N$, so $a = g + \dfrac{N}{m}$ (an equation written without that statement does not earn this point) **1 point**

For noting that the gravitational force does not change between experiments **1 point**

For reasoning that halving N halves only the panel's contribution to the acceleration **1 point**

For concluding that the new acceleration exceeds half the original because the g term is not halved (for example, $g + \dfrac{N}{2m}$ exceeds $\dfrac{g}{2} + \dfrac{N}{2m}$ by $\dfrac{g}{2}$) **1 point**

For a response that is logical, follows from start to finish, and contains no incorrect or irrelevant statements **1 point**

Example response: Two forces act on the block, both downward: its weight and the push from the panel. By Newton's second law, the acceleration is the sum of these forces divided by the block's mass, $a = g + N/m$. In the second experiment the weight is unchanged and only the panel's push is halved, so only the panel's share of the acceleration is halved. The weight's share, g, stays whole, so the new acceleration is larger than half the original one.

Points 3 and 4 can be earned together by a correct algebraic comparison that keeps the g term whole and halves only N/m, for example $g + \frac{N}{2m}$ versus $\frac{1}{2}\left(g + \frac{N}{m}\right)$.

Trap: "Equal to half, because acceleration is proportional to force." Acceleration is proportional to the *net* force, and halving one of two forces does not halve the sum.

(b) **1 point**

For selecting "Less than twice" and correctly explaining that doubling the panel's force doubles only the panel's contribution to the acceleration, N/m, while the gravitational contribution, g, is unchanged, so the new acceleration $g + \dfrac{2N}{m}$ is less than twice the original, $2g + \dfrac{2N}{m}$ (no credit for this part without the correct selection) **1 point**

Question 3.4 (Testing the equivalence principle, page 17) — 7 points

(a) — **3 points**

For a measurement of the gravitational mass: hang the cart at rest from the force probe and record the probe's reading F_{hang} — **1 point**

For a measurement of the inertial mass: pull the cart horizontally along the floor with the force probe (held level), recording the probe's reading F while the motion detector records the cart's acceleration a — **1 point**

For a way to reduce uncertainty, for example repeating each measurement, or pulling with several different forces and graphing — **1 point**

(b) — **2 points**

For finding the inertial mass, $m_i = F/a$ for each trial (or the slope of a graph of F against a) — **1 point**

For finding the gravitational mass, $m_g = F_{\text{hang}}/g$, and comparing it with m_i, deciding that the principle is satisfied if the two agree within the uncertainty (for example the spread of repeated trials) — **1 point**

(c) — **2 points**

For explaining the hanging measurement: the cart is at rest, so by Newton's second law the probe's upward force balances the gravitational force, $F_{\text{hang}} = m_g g$ — **1 point**

For explaining the horizontal measurement: the floor is horizontal and the wheels are frictionless, so the probe's force is the net horizontal force, and Newton's second law gives $F = m_i a$ — **1 point**

Trap: Using only one kind of measurement, for example weighing the cart and calling the result "the mass." The test needs one measurement that depends on gravity and one that does not.

Question 3.5 (Half-rough table, page 18) — 7 points

Answers may use $g = 9.8$ or 10 m/s^2.

(a) — **2 points**

For identifying the forces along the direction of motion for the block–string–mass system on the rough portion, the hanging mass's weight mg forward and kinetic friction $\mu_K mg = 2mg$ backward, and applying Newton's second law to the whole system with its total mass $2m$: $(2m)a = mg - 2mg$; or for separate Newton's second law equations for the block ($ma = T - 2mg$) and the hanging mass ($ma = mg - T$) solved together — **1 point**

For $|a| = \dfrac{g}{2} = 4.9$ m/s^2 (5 m/s^2 with $g = 10$ m/s^2) — **1 point**

Trap: Using only the block's mass m in Newton's second law, which gives g. The hanging mass must be slowed down too.

(b) — **2 points**

For using constant-acceleration kinematics with the deceleration from (a), $v_D^2 = v^2 - 2|a|D$, or for showing that the stopping distance $\dfrac{v^2}{2|a|} \approx 10$ m exceeds $D = 7.5$ m, so the block reaches the frictionless portion — **1 point**

For $v_D = \sqrt{v^2 - gD} = 5.1$ m/s (5.0 m/s with $g = 10$ m/s^2) — **1 point**

Score (b) for consistency with the acceleration found in (a).

(c) — **2 points**

For reasoning that on the frictionless portion the only unbalanced force on the block–string–mass system is the hanging mass's weight, mg, acting on the total mass $2m$, so $|a| = \dfrac{g}{2} = 4.9$ m/s^2 — **1 point**

For stating that the block speeds up, because that net force on the system points in the direction of the block's motion — **1 point**

(d) — **1 point**

For selecting C and correctly explaining that the acceleration is constant on each portion, so the graph is made of two straight segments; the speed decreases on the rough portion but does not reach zero before the frictionless portion (b), and then increases (equally steep segments, since both accelerations have magnitude $\frac{g}{2}$, are not required) (no credit for this part without the correct selection) **1 point**

Trap: Choosing D: "friction ($2mg$) is greater than the hanging weight (mg), so the block stops and static friction holds it." That would happen only if the block stopped on the rough portion; check the distance first.

Question 3.6 (Block, planks, and a leading ball, page 20) 12 points

(a)(i) **2 points**

For reasoning about the sphere: it has no vertical acceleration, so the vertical component of the string's tension on it balances its weight, $T\sin\theta = Mg$ **1 point**

For reasoning about the block: the massless string pulls down on the block with that same vertical component, Mg, and the block has no vertical acceleration, so the planks' upward normal force balances the block's weight plus that pull, $N = mg + Mg = (m + M)g$ **1 point**

A system argument earns both points: no part of the block–string–sphere system accelerates vertically, and the only vertical external forces on it are its total weight, $(m + M)g$, and the planks' normal force, so the two balance.

(a)(ii) **1 point**

For selecting "Stay the same" and correctly explaining that the only horizontal external force on the system is kinetic friction, proportional to the normal force $(m + M)g$, so both the stopping force and the system's inertia are proportional to $m + M$; doubling M scales both by the same factor and leaves a unchanged (no credit for this part without the correct selection) **1 point**

Trap: "Increase" or "decrease" from counting only the block's weight in the normal force. The string transmits the sphere's weight to the planks.

(a)(iii) **1 point**

For selecting "Stay the same" and correctly explaining that, for the sphere, the vertical component of the tension balances the sphere's weight while the horizontal component gives it the acceleration a; both are proportional to M, so the string's direction depends only on the ratio of a to g, which is unchanged by (a)(ii) (no credit for this part without the correct selection) **1 point**

(b)(i) **2 points**

For correct Newton's second law equations, including the normal force on the block $N = mg + T\sin\theta$ (with $T\sin\theta = Mg$ for the sphere), or the system equation $(m + M)a = \mu_K N$ with $N = (m + M)g$ **1 point**

For $a = \mu_K g$ **1 point**

(b)(ii) **2 points**

For the sphere's equations $T\sin\theta = Mg$ and $T\cos\theta = Ma$, giving $\tan\theta = \frac{g}{a}$ **1 point**

For $\tan\theta = \frac{1}{\mu_K}$ (so $\theta = \tan^{-1}\frac{1}{\mu_K}$) **1 point**

(c)(i) **1 point**

For using $\tan\theta = \frac{1}{\mu_K}$ (or the student's own result from (b)(ii)) with the measured angles to get $\mu_K \approx 0.78$ for Group 1 (52°) and $\mu_K \approx 0.55$ for Group 7 (61°) **1 point**

(c)(ii) **1 point**

For selecting "Yes" and correctly justifying it with the uncertainty: the angle ranges 50°–54° and 59°–63° do not overlap, so the corresponding ranges of μ_K (about 0.73–0.84 and 0.51–0.60) do not overlap either, and the difference is larger than the measurement uncertainty (no credit for this part without the correct selection) **1 point**

(c)(iii) **1 point**

For selecting "No" and correctly explaining that, by (b), neither a nor θ depends on m: removing mass from the block reduces the friction force and the system's inertia in the same proportion (no credit for this part without the correct selection) **1 point**

(c)(iv) **1 point**

For a mechanism that lowers μ_K from group to group (for example the planks or the block's surface wearing smoother with use, or picking up wax, dust, or moisture), tied to the data and to (b): the values from (c)(i) show μ_K falling over the class period, and by $\tan\theta = 1/\mu_K$ a falling μ_K gives a rising θ (a mechanism stated without this support earns no credit) **1 point**

The student's last step (a smaller acceleration gives a more vertical string) is correct, by $\tan\theta = g/a$*; only the premise that removing mass from the block lowers the acceleration is wrong. A response that says so earns the point in (c)(iii). Across all seven groups the data correspond to* μ_K *falling from about* 0.8 *to about* 0.5*. Score (c) for consistency with the expressions the student derived in (b).*

Question 3.7 (Testing a friction claim, page 23) 12 points

(a) **3 points**

For a setup that controls the normal force, for example the block on the horizontal board with known masses stacked on it (normal force = total weight), or the block pressed against the board by a measured force, or the block on the board with the board tilted **1 point**

For a way to measure the largest friction force before slipping, for example a spring scale or force probe pulling the block horizontally, or raising one end of the tilted board slowly until the block just slips and measuring that angle θ_c with a protractor (or the board's rise and length with a meterstick) **1 point**

For a labeled diagram consistent with the description **1 point**

(b) **3 points**

For measuring the normal force (balance for the masses, or a force probe) for each trial, over a range wide enough that the claimed 8.0 N limit would clearly be reached (normal forces large enough that, without a limit, friction would clearly exceed 8.0 N) **1 point**

For measuring $f_{S,\max}$: increase the pulling force slowly and record the largest reading just before the block starts to slide (or, with the tilting board, raise the board slowly and record the angle θ_c at which the block starts to slide, varying the normal force by adding masses to the block) **1 point**

For a way to reduce uncertainty, for example repeating each trial and averaging, or pulling more slowly to catch the peak reading **1 point**

(c) **2 points**

For graphing $f_{S,\max}$ against the normal force (or computing their ratio for each trial) **1 point**

For a decision rule that uses the uncertainty: the claim is supported if the points rise along a line through the origin and then level off at 8.0 N (within error bars); it is contradicted if, within uncertainty, points above the leveling-off region exceed 8.0 N or keep rising **1 point**

(d) **2 points**

For explaining that when the block is on the verge of slipping, the horizontal forces balance, so the largest pulling force before slipping equals $f_{S,\max}$ (Newton's second law with zero acceleration); or, for the tilting board, that the forces along the board balance just before slipping, so $f_{S,\max} = mg\sin\theta_c$ **1 point**

For explaining how the normal force is known from the measurement: with the board horizontal and nothing else pushing vertically, it equals the weight of the block and added masses (vertical forces balance); or, if the block is pressed against the board, it equals the measured pressing force (forces perpendicular to the board balance); or, for the tilting board, the forces perpendicular to the board balance, so $N = mg\cos\theta_c$ **1 point**

(e) **1 point**

For selecting "No" and correctly justifying it with the data: at least one measured maximum static friction force is above 8.0 N by more than the ± 0.1 N uncertainty (for example 8.9 N at 12.00 N, or 15.1 N at 20.00 N), which the claim forbids (no credit for this part without the correct selection) **1 point**

(f) **1 point**

For $\mu_S \approx 0.75$ (accept 0.73 to 0.80), found as the slope of maximum static friction force against normal force or from the ratio $f_{S,\max}/N$ for any row **1 point**

The best-fit slope is 0.750. *The first row alone gives* 0.80, *which is accepted; that row has the largest relative uncertainty.*

Question 3.8 (Up or down the ramp, page 25) 7 points

(a) **1 point**

For selecting "The same" and correctly explaining that the only forces are gravity and the normal force, only gravity has a component along the plane ($mg\sin\theta$, down the plane), and neither force depends on the direction of motion, so by Newton's second law the acceleration ($g\sin\theta$ down the plane) is the same either way (no credit for this part without the correct selection) **1 point**

Trap: "Greater sliding down, because gravity helps." Sliding up, gravity slows the block at the same rate it speeds the block up sliding down.

(b) **4 points**

Correct selection: "Greater sliding up." No point is awarded for the selection itself.

For reasoning that kinetic friction opposes the motion: down the plane while the block slides up, up the plane while it slides down, while the component of gravity along the plane is down the plane both times **1 point**

For comparing the net forces along the plane: sliding up, the gravity component and friction point the same way and add, $mg\sin\theta + f$; sliding down, they point opposite ways, so the net force, $|mg\sin\theta - f|$, is smaller **1 point**

For concluding with Newton's second law that the larger net force, while sliding up, gives the larger acceleration **1 point**

For a response that is logical, follows from start to finish, and contains no incorrect or irrelevant statements **1 point**

(c) **2 points**

For Newton's second law along the plane with the normal force $mg\cos\theta$ and kinetic friction $\mu_K mg\cos\theta$ pointing down the plane along with $mg\sin\theta$, giving a deceleration of magnitude $g(\sin\theta+\mu_K\cos\theta)$, or the equivalent work–energy equation, $-(mg\sin\theta + \mu_K mg\cos\theta)\,d = 0-\frac{1}{2}mv_0^2$ **1 point**

For $d = \dfrac{v_0^2}{2g(\sin\theta + \mu_K\cos\theta)}$, from $0 = v_0^2 - 2ad$ (or from the work–energy theorem) **1 point**

Trap: Writing the friction force as $\mu_K mg$. On the incline the normal force is $mg\cos\theta$.

Question 3.9 (Box on an accelerating wedge, page 27) 7 points

(a) **6 points**

Correct selection: "Less than doubled." No point is awarded for the selection itself.

For identifying the forces on the box: gravity and the normal force (perpendicular to the plane), and stating that the box must have the same horizontal acceleration a as the plane and no vertical acceleration **1 point**

For the vertical balance, $N\cos\theta = mg$ **1 point**

For the horizontal equation, $N\sin\theta = ma$ **1 point**

The two points above can also be earned together by a correct force-vector triangle (gravity plus normal force adding to $m\vec{a}$, horizontal) that leads to $\tan\theta = a/g$.

For combining them: $\tan\theta = \dfrac{a}{g}$, so doubling a doubles $\tan\theta$ **1 point**

For reasoning that $\tan\theta$ grows faster than in proportion to θ (for example $\tan 60^\circ \approx 1.73$ is more than twice $\tan 30^\circ \approx 0.58$), so doubling $\tan\theta$ requires less than doubling θ **1 point**

For a response that is logical, follows from start to finish, and contains no incorrect or irrelevant statements **1 point**

Trap: "Exactly doubled, because a is proportional to θ." The relationship is through $\tan\theta$, which is not proportional to θ.

(b) **1 point**

For selecting "Greater than" and correctly explaining that the vertical component of the normal force must still balance the box's weight, $N\cos\theta = mg$, and the angle is larger in the second experiment, so $\cos\theta$ is smaller and N is larger (or: the normal force must now supply a larger horizontal component, ma, as well as the same vertical component, mg, so $N = m\sqrt{g^2 + a^2}$ is larger) (no credit for this part without the correct selection) **1 point**

Question 3.10 (Drag and a speed graph, page 28) 7 points

(a) **6 points**

Correct selection: "No." No point is awarded for the selection itself.

For stating that the slope of a speed–time graph (for straight-line motion) is the acceleration **1 point**

For Newton's second law for the falling object: $ma = mg - F_{\text{DRAG}}$, with the gravitational force constant **1 point**

For reasoning that after t_C the slope is constant and positive, so the acceleration, and therefore the drag force, is constant **1 point**

For noting that the speed keeps increasing after t_C (the graph is still rising) **1 point**

For concluding that the graph shows an interval in which the speed increases while the drag force does not, which contradicts the claim (the earlier, concave-down portion, in which drag grows as speed grows, does not rescue a claim about "whenever") **1 point**

For a response that is logical, follows from start to finish, and contains no incorrect or irrelevant statements **1 point**

Trap: "Yes: the graph curves over at first, which shows drag growing as speed grows." The claim says "whenever"; one interval where it fails is enough to contradict it.

(b) **1 point**

For selecting "Greater than" and correctly explaining that the slope of the graph (the acceleration) at $\frac{1}{2}t_C$ is greater than the constant slope after t_C, because the graph is concave down before t_C and its slope decreases to the straight-line value; since $mg - F_{\text{DRAG}} = ma$ with mg constant, the smaller acceleration at $2t_C$ means the larger drag force (no credit for this part without the correct selection) **1 point**

Question 3.11 (Boxcar on a ramp, page 29) 7 points

(a) **1 point**

For selecting $\phi = \theta$ and correctly explaining that the boxcar–ball system, acted on only by gravity and the ramp's normal force, accelerates down the ramp at $g\sin\theta$; the ball has that same acceleration, and its gravitational force alone supplies exactly $mg\sin\theta$ along the ramp, so the tension has no component along the ramp: the string is perpendicular to the ramp, which tilts it from the vertical by θ (no credit for this part without the correct selection) **1 point**

Trap: "The string hangs straight down ($\phi = 0$, so less than θ)": true only if the boxcar's velocity were constant.

(b) **4 points**

Correct selection: $\phi' > \theta$. No point is awarded for the selection itself.

For reasoning that, sliding up the ramp, the boxcar has both the down-ramp component of gravity and kinetic friction (opposing its motion, so down the ramp) acting on it, so its acceleration down the ramp is greater than $g\sin\theta$ **1 point**

For reasoning that the ball, at rest relative to the boxcar, needs that same, larger acceleration, which its gravitational force alone cannot supply along the ramp, so the tension must have a component down the ramp **1 point**

For concluding from this that the string tilts past the perpendicular to the ramp, farther from the vertical, so $\phi' > \theta$ **1 point**

For a response that is logical, follows from start to finish, and contains no incorrect or irrelevant statements **1 point**

Trap: "Cannot be determined without knowing the friction coefficient." The size of the friction force doesn't matter, only its direction.

(c) **2 points**

For Newton's second law along the ramp for the boxcar–ball system (total mass M_{tot}), with the normal force $M_{\text{tot}}\, g \cos\theta$ (the system has no acceleration perpendicular to the ramp) and kinetic friction $\mu_K M_{\text{tot}}\, g \cos\theta$ pointing down the ramp along with $M_{\text{tot}}\, g \sin\theta$ **1 point**

For $a = g(\sin\theta + \mu_K \cos\theta)$ **1 point**

Chapter 4: Circular motion and gravitation

Question 4.1 (Binary stars, page 31) 7 points

(a) **3 points**

For reasoning that the gravitational force on each star provides the net inward force and is proportional to the product of the two masses, so it grows by the square of the factor **1 point**

For reasoning that the inertia of each star grows only by the factor itself, so the inward acceleration v^2/r is larger, and with the orbit radius r unchanged a larger acceleration requires a larger speed **1 point**

For a logical, relevant paragraph that connects these ideas to the conclusion that the speed increases, without contradictory or incorrect statements **1 point**

The answer blanks are not scored. An equation-based paragraph earns the first two points if it sets the gravitational force equal to $m\frac{v^2}{r}$ and explains that one factor of m (the orbiting star's inertia) cancels while the other (the attracting star's mass) remains, so v^2 is proportional to m at fixed ℓ. Quoting a formula for v without that reasoning earns neither.

Example response: The force each star feels is $Gm \cdot m/\ell^2$, so tripling both masses makes it nine times as large, while each star's mass is only three times as large. The inward acceleration triples, and with the same orbit radius that takes a faster speed.

Trap: "Remain the same: mass cancels, as it does for a satellite." The mass cancels only for the orbiting object's own mass; here the attracting mass also increased.

(b) **4 points**

For using Newton's second law for circular motion: $m\dfrac{v^2}{r} = F_{\text{net, inward}}$ **1 point**

For the gravitational force with separation ℓ: $F = \dfrac{Gm^2}{\ell^2}$ **1 point**

For the orbit radius $r = \dfrac{\ell}{2}$ (not ℓ) **1 point**

For $v = \sqrt{\dfrac{Gm}{2\ell}}$ **1 point**

Trap: Using ℓ for both the separation and the orbit radius gives $\sqrt{Gm/\ell}$, which earns 2 of 4 (the orbit-radius and final-answer points are lost). Using $\ell/2$ in both places gives $\sqrt{2Gm/\ell}$, which also earns 2 of 4 (the force and final-answer points are lost).

Question 4.2 (Four moons, page 32) **12 points**

(a) **4 points**

For choosing quantities whose relationship is linear under the claim: T^2 and R^3 (or equivalent, for example T^2/R^3 for each moon) **1 point**

For a decision rule: the claim is supported if the points fall on a straight line through the origin (or if T^2/R^3 is the same for every moon) **1 point**

For a method of estimating the uncertainty in the computed quantities, for example recomputing T^2 and R^3 with T and R shifted by their stated uncertainties **1 point**

For using those uncertainties in the decision, for example error bars that a line through the origin must pass through, or ranges of T^2/R^3 that must overlap **1 point**

(b) **3 points**

For Newton's second law for each moon in uniform circular motion, with the only significant inward force being the planet's gravity: $\dfrac{mv^2}{R} = \dfrac{GMm}{R^2}$ **1 point**

For $v = \dfrac{2\pi R}{T}$ **1 point**

For combining to $T^2 = \dfrac{4\pi^2}{GM}R^3$ and explaining that, if M is the same for every moon (all the mass is at the center), T^2 versus R^3 is a line through the origin with slope $\frac{4\pi^2}{GM}$ **1 point**

(c) **5 points**

For correct values of the chosen quantities, for example $T^2/R^3 \approx 2.97 \times 10^{-16}$, 2.96×10^{-16}, 1.48×10^{-16}, 1.48×10^{-16} $\mathrm{s^2/m^3}$ for A–D (or $R^3 \approx 2.38\times10^{26}$, 7.79×10^{26}, 1.23×10^{27}, 1.73×10^{27} $\mathrm{m^3}$ and $T^2 \approx 7.08 \times 10^{10}$, 2.30×10^{11}, 1.81×10^{11}, 2.56×10^{11} $\mathrm{s^2}$) **1 point**

For a correctly plotted graph or table consistent with the method in (a) **1 point**

For uncertainty estimates carried out as described in (a) (for example, moon A's ratio ranges from about 2.5 to 3.5×10^{-16}, moon C's from about 1.35 to 1.6×10^{-16}) **1 point**

For observing that A and B agree with each other and C and D agree with each other, but the two pairs differ by about a factor of two, far more than the uncertainty **1 point**

For concluding that the data are inconsistent with the claim **1 point**

Points 2–5 are scored for consistency with the student's method in (a), even if that method earned less than full credit, as long as the method can test the claim. A student may also note that the two pairs are consistent with about twice as much mass inside the orbits of C and D as inside the orbits of A and B; this is welcome but not required.

Trap: Fitting the four points with a line that is not forced through the origin, or a line that misses two points, and calling the fit "close enough" without using the uncertainties. Moons C and D sit at about half the slope of A and B.

Chapter 5: Impulse and momentum

Question 5.1 (Cart and spring bumper, page 34) — 12 points

(a) — **4 points**

For measuring the mass of the cart (with the bars added) with a balance, and changing the mass between trials by adding bars — **1 point**

For a way to give the cart the same speed just before the bumper in every trial, and to check it. Examples: release the cart from the same mark on a ramp that leads onto the track (the speed at the bottom does not depend on mass); or push the cart and keep only trials in which a motion detector reads the target speed within a stated tolerance — **1 point**

For measuring the velocities just before and just after the bounce with a motion detector (or photogates, or video analysis), so that impulse can be found from the change in momentum — **1 point**

For repeating trials at each mass, or using several masses, to reduce uncertainty — **1 point**

A procedure that launches every cart with the same compressed spring does not *earn the second point: a fixed launcher gives lighter carts higher speeds. A procedure that uses a force sensor earns no credit for that step, because the problem says none is available.*

(b) — **2 points**

For calculating the impulse in each trial from $J = m\,|\Delta v|$, with the velocity change taken as the sum of the speeds before and after, because the direction reverses — **1 point**

For a graph of impulse versus mass and a correct decision rule: if the claim is true the points lie on a horizontal line; if the impulse grows with mass, the claim is false — **1 point**

(c) — **2 points**

For axes labeled with quantities and units, scales that use more than half of the grid, and all five points plotted correctly — **1 point**

For a straight line with points on both sides, close to passing through the origin (slope about 1.4 N·s/kg) — **1 point**

(d) — **1 point**

For answering "No" and correctly explaining, by reference to the graph, that the impulse increases in proportion to the mass (the best-fit line rises and passes close to the origin) rather than staying constant as the claim requires (no credit for this part without the correct answer) — **1 point**

(e) — **3 points**

For applying the impulse–momentum theorem with directions: $J = m\,(v_f - v_i)$ with $v_i = -0.80$ m/s and v_f positive, so $J = m\,(0.80\ \text{m/s} + v_f)$ — **1 point**

For identifying the slope of the impulse-versus-mass line as $0.80\ \text{m/s} + v_f$ — **1 point**

For a speed consistent with the student's line, about 0.60 m/s (accept 0.55–0.65 m/s) — **1 point**

Trap: Subtracting speeds instead of velocities: slope $= v_f - 0.80$, so $v_f \approx 2.2$ m/s. That is faster than the approach speed, which a passive spring bumper cannot produce; a student who checks the result should catch it. Earns 1 of 3 (slope point only).

Question 5.2 (Spring between two blocks, page 36) — 7 points

(a) — **5 points**

For reasoning that in Experiment 1 the momentum of the A–B system is constant, because the net external horizontal force on the system is zero (frictionless surface; gravity and the normal forces are vertical) — **1 point**

For reasoning that at greatest compression in Experiment 1 the blocks move with the same velocity, which is not zero (it is $\frac{v}{2}$), so some kinetic energy remains — **1 point**

For reasoning that in both experiments the mechanical energy of the system is constant, so the energy stored in the spring at greatest compression equals the initial kinetic energy minus any kinetic energy remaining; in Experiment 2, block A is momentarily at rest at greatest compression, so all of its kinetic energy is stored — **1 point**

For concluding from this energy comparison that the spring stores less energy at greatest compression in Experiment 1 than in Experiment 2, and that because the stored energy increases with compression for the same spring, the greatest compression is less in Experiment 1 — **1 point**

For a logical, relevant argument that connects these ideas to the answer without contradictory or incorrect statements — **1 point**

The answer blanks are not scored. A paragraph that reaches "less than" by correct reasoning earns the conclusion point even if a different blank is checked, but the contradiction costs the last point.

Example response: Take blocks A and B and the spring as the system. In Experiment 1, the only horizontal forces are between A, the spring, and B, so the system's momentum stays mv. The spring is most compressed when A and B are no longer approaching each other, which means they have the same velocity; equal masses sharing momentum mv each move at $\frac{v}{2}$. No friction acts, so the total energy is unchanged, and the spring stores only the kinetic energy that the blocks no longer have: $\frac{1}{2}mv^2 - \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2$. In Experiment 2 the glue exerts an external force on B, so momentum is not conserved, but no work is done on the system because B does not move. Block A stops at greatest compression, so the spring stores all of A's kinetic energy, $\frac{1}{2}mv^2$. The spring stores less energy in Experiment 1, so it compresses less.

Trap: "The same, because the same energy goes in." That reading ignores the kinetic energy the moving pair keeps in Experiment 1. A second trap is claiming momentum is conserved in Experiment 2 as well; that statement is incorrect and loses the last point even when the rest of the argument is correct.

(b) — **2 points**

For relating the stored energies: $\frac{1}{2}kx_1^2 = \frac{1}{4}mv^2$ and $\frac{1}{2}kx_2^2 = \frac{1}{2}mv^2$ — **1 point**

For the correct ratio $\dfrac{x_1}{x_2} = \dfrac{1}{\sqrt{2}} \approx 0.71$ — **1 point**

Trap: Giving $\frac{1}{2}$, the ratio of the energies, rather than of the compressions; earns 1 of 2.

Question 5.3 (Charlie's momentum formula, page 37) — 7 points

(a) — **6 points**

Correct selection: "Greater than." No point is awarded for the selection itself. Charlie gets $v/\sqrt{2} \approx 0.71v$; the correct answer is $v/2$.

For reasoning that the two-cart system's momentum is conserved, because the track is frictionless and horizontal, so no net external horizontal force acts — **1 point**

For the correct result: with $p = mv$, doubling the moving mass halves the speed, $V = v/2$ — **1 point**

For Charlie's result: with $\frac{1}{2}mv^2$ held constant, doubling the mass halves v^2, so the speed is divided only by $\sqrt{2}$, $V_C = v/\sqrt{2}$ — **1 point**

For recognizing that Charlie's quantity, $\frac{1}{2}mv^2$, is kinetic energy, which is not conserved in a collision in which the carts stick together, so Charlie's calculation rests on a false conservation statement — **1 point**

For identifying why the results differ: the conserved quantity is proportional to v, so sharing it with twice the mass halves v, while Charlie's quantity is proportional to v^2, so the speed falls only by a factor of $\sqrt{2}$ — **1 point**

For a response that is logical, follows from start to finish, and contains no incorrect or irrelevant statements **1 point**

Trap: "Equal: either way the momentum is conserved and the mass doubles." The two formulas respond differently to doubling the mass.

(b) **1 point**

For selecting "Less than" and correctly explaining that momentum conservation gives the stuck carts the speed $v/2$, so their kinetic energy is $\frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2$, half of the initial $\frac{1}{2}mv^2$ (or: keeping the kinetic energy unchanged would require the speed $v/\sqrt{2}$ found by Charlie's method, which is greater than the actual speed $v/2$) (no credit for this part without the correct selection) **1 point**

Question 5.4 (Horizontal momentum and a pulley, page 38) 7 points

For stating the impulse–momentum relationship for the system: the change in the system's horizontal momentum equals the horizontal component of the net external impulse **1 point**

For identifying the pulley, which is outside the system, as exerting a force on the string **1 point**

For the direction of that force: the two taut segments of string pull the pulley left and down, so (Newton's third law) the pulley pushes the string right and up; it has a horizontal component to the right **1 point**

For concluding that this force, acting over the time interval, delivers a horizontal impulse to the right, which accounts for the block's increasing momentum to the right (the slotted mass set has no horizontal momentum) **1 point**

For identifying the flaw in the students' argument: the gravitational force being necessary for the motion does not make it the only external force; the change in momentum equals the impulse of the *net* external force **1 point**

For noting that the other external forces (gravity on the block and slotted mass set, the table's normal force) are vertical, so the pulley's force is the only external force with a horizontal component **1 point**

For a response that is logical, follows from start to finish, and contains no incorrect or irrelevant statements **1 point**

Example response: The horizontal momentum of the system changes by the horizontal impulse of the net external force on it. Gravity on the slotted mass set is not the only external force: the table pushes up on the block, gravity pulls down on the block, and the pulley pushes on the string where the string bends around it. Because the string turns from horizontal to vertical, the pulley pushes the string up and to the right. That rightward push, acting over time, is a horizontal impulse, and it is what gives the block its growing momentum to the right. Gravity on the hanging mass is needed to make the string pull at all, but it is the pulley's force that supplies the horizontal impulse.

Trap: Saying "the tension pulls the block" and stopping there. Tension is internal to this system; the impulse must come from outside it. (A student who reaches the pulley's push as the reaction to the string's pull on the pulley has given a valid argument.)

Chapter 6: Center of mass

Question 6.1 (Carts pushed apart by magnets, page 39) — 7 points

(a) — **2 points**

For using the definition of center of mass with positions measured from cart 1: $x_{\text{cm}} = \dfrac{m(0) + 3m(d)}{m + 3m}$ — **1 point**

For $x_{\text{cm}} = \dfrac{3d}{4}$ (toward cart 2) — **1 point**

Trap: Writing $d/4$: weighting the distance by the wrong cart's mass puts the center of mass closer to the lighter cart.

(b)(i) — **1 point**

For identifying a correct aspect with a correct reason: the forces are equal in size (Newton's third law), or cart 1 speeds away faster (same size force, smaller mass, larger acceleration, for the same time), or cart 1 moves farther (three times as far) — **1 point**

(b)(ii) — **2 points**

For identifying the incorrect aspect: the claim that the center of mass moves — **1 point**

For explaining that the magnetic forces are internal to the two-cart system and the net external horizontal force is zero, so the system's momentum stays zero and the center of mass stays at rest; cart 1 moving farther is exactly balanced by its smaller mass — **1 point**

Example response: The pushes between the magnets are internal to the two-cart system, and nothing outside pushes horizontally (friction is negligible), so the system's total momentum stays zero. The center of mass started at rest, so it stays put. Cart 1 does move three times as far, but it has one-third the mass, so $m\,\Delta x_1 + 3m\,\Delta x_2 = 0$.

Trap: Agreeing with the claim because cart 1 "moves more." The center of mass weights each cart's displacement by its mass.

(c) — **2 points**

For using a fixed center of mass (or zero total momentum at every instant) to relate the displacements: $ms = 3m\,s_2$ — **1 point**

For $s_2 = \dfrac{s}{3}$ — **1 point**

Question 6.2 (Air-hockey pucks, page 40) — 12 points

(a) — **3 points**

For recording video of the pucks before, during, and after the interaction, with the meter stick in view for scale, and reading both pucks' positions from frames at known times (from the frame rate) — **1 point**

For computing the position of the center of mass at each of those times from the two positions and the masses, $x_{\text{cm}} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$ (the masses may be taken as given or measured with the balance) — **1 point**

For a test of the claim: graphing x_{cm} against time (or computing its velocity over successive equal intervals) and checking whether it is one straight line (constant velocity) through the times before, during, and after the interaction — **1 point**

Ways to reduce uncertainty (many frames, several trials, camera directly overhead) are good practice but are not required.

(b)(i) — **1 point**

For the center-of-mass position $x_{\text{cm}} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$ with correct values: 0.310, 0.350, 0.391, 0.430, 0.470, 0.510, 0.550, 0.590, 0.630 m (also accept $m_1x_1 + m_2x_2$, which is proportional to x_{cm}: 0.155, 0.175, …, 0.315 kg·m) — **1 point**

(b)(ii) — **2 points**

For correctly scaled and labeled axes (with units) that use more than half the grid — **1 point**

For correctly plotted points — **1 point**

(b)(iii) **1 point**

For selecting "Yes" and correctly explaining that the points lie on one straight line (constant slope), including the points at $t = 0.8$ s and 1.0 s, when the magnets were pushing, and that a straight x–t graph means constant velocity (no credit for this part without the correct selection) **1 point**

(c) **2 points**

For the slope of a best-fit line, $v_{\text{cm}} \approx 0.20$ m/s (accept 0.19–0.21 m/s), not computed from two data points far from the line **1 point**

For the prediction $\dfrac{m_1 v_1}{m_1 + m_2}$ with $v_1 \approx 0.50$ m/s (from the first rows), giving 0.20 m/s, and noting the agreement **1 point**

(d)(i) **2 points**

For computing the group's quantity, $(0.30x_1 + 0.20x_2)/0.50$, at the needed times (0.240 and 0.420 m at 0.0 s and 0.6 s; 0.485 and 0.544 m at 1.0 s and 1.6 s), or, equivalently, from the pucks' velocities in each interval **1 point**

For slopes of about 0.30 m/s before the interaction and about 0.10 m/s after it (accept ± 0.02 m/s) **1 point**

Equivalent route: the group's quantity equals the true x_{cm} minus $0.2\,(x_2 - x_1)$; the separation shrinks at 0.50 m/s before and grows at 0.50 m/s after, giving $0.20 + 0.10$ and $0.20 - 0.10$ m/s.

(d)(ii) **1 point**

For selecting "No" and correctly explaining that during the interaction the magnets change the pucks' velocities in such a way that $m_1\Delta v_1 + m_2\Delta v_2 = 0$ (momentum conservation), so only the average weighted by the correct masses has a velocity that does not change; with the masses swapped, puck 1's velocity change is overweighted, so the slope changes (from about 0.30 to 0.10 m/s) and the graph is two straight segments, not one line (no credit for this part without the correct selection) **1 point**

Citing only the two different slopes from (d)(i), with no reason why the correct masses avoid the change, does not earn the point.

Trap: "Yes: each puck moves at constant velocity before and after, so any weighted average does too." Each segment is straight, but the two segments have different slopes; only the correct masses make them match.

Chapter 7: Work and energy

Question 7.1 (Same push, different blocks, page 42) **7 points**

Correct selection: $v_1 > v_2$. The question asks for energy principles; a Newton's-second-law argument, however correct, does not earn the energy points.

For identifying the energy principle (at least by name): the work done on the block equals its change in energy. Writing $F\Delta\ell = \frac{1}{2}mv^2$ alone does not earn this point, because that equation can come from kinematics without any energy principle. **1 point**

For stating that kinetic energy is $K = \frac{1}{2}mv^2$ (not merely writing $\frac{1}{2}mv^2$ somewhere) **1 point**

For using the principle with zero initial kinetic energy and no internal-energy change (frictionless) to conclude that the work done equals the final kinetic energy **1 point**

For stating that work is force along the displacement times the distance, $W = F\Delta\ell$ **1 point**

For concluding that the work, and so the final kinetic energy, is the same in both experiments because F and $\Delta\ell$ are the same **1 point**

For concluding that the same kinetic energy with three times the mass requires a smaller speed **1 point**

For a response that is logical, follows from start to finish, and contains no incorrect or irrelevant statements **1 point**

Example response: Take the block as the system. By the work–energy principle, the work done on it equals its change in energy. The track is frictionless, so no thermal energy is produced, and the block starts at rest, so all of the work becomes kinetic energy, $K = \frac{1}{2}mv^2$. Work is the force along the motion times the distance, $F\Delta\ell$, and both are the same in the two experiments, so both blocks end with the same kinetic energy. The second block has three times the mass, so to have the same kinetic energy it must be moving more slowly: $v_1 > v_2$.

A response that does not say the initial kinetic energy is zero but argues that it is the same in both experiments can earn the third point.
Trap: "Same force, same distance, so same speed." Same work gives the same kinetic energy, not the same speed.

Question 7.2 (Toys in slots, page 43) — 12 points

(a)(i) — **1 point**

For reasoning, by conservation of mechanical energy for the ball–Earth system (frictionless, so no thermal energy; the normal force does no work), that the ball's kinetic energy at x_2, and so its speed there, depends only on its height there, and that the greater the height, the smaller the speed — **1 point**

(a)(ii) — **2 points**

For reasoning that the ball must stay in contact, so each toy must start at the height where the ball arrives: a step down launches the ball, and a step up is a wall. The first toy must start at h_{PL} (A, B, or D), and the second must start at the height where the first ends — **1 point**

For reasoning that the greatest ending height, H, is reached only by A then E or by D then A (B ends at h, and E cannot be reached from the platform), and both of these use Toy A — **1 point**

Trap: Choosing Toy E, the tallest toy, without checking that the ball can get onto it. From the platform, E is a wall.

(a)(iii) — **1 point**

For selecting Toy D in Slot 1 and Toy A in Slot 2 and correctly explaining that the climb up A takes the same time in either order (it starts at h_{PL} with speed v_0 both times), so the difference is the level slot: D–A crosses it on D at the full speed v_0, while A–E crosses it on E at height H, where the ball is slowest; so D–A takes less time (no credit for this part without the correct selection) — **1 point**

Equivalent explanation: a pointwise comparison of horizontal velocities that pairs corresponding points (the two climbs up A match point for point; on the level toy the ball moves at v_0 on D but more slowly on E). A comparison of speeds at the same x is not enough, because on the curved toy only the horizontal part of the velocity sets the crossing time.

(b) — **1 point**

For selecting "Consistent" and correctly explaining that the graph shows U_G increasing with height, which is what the reasoning in (a) used (higher means more potential energy and less kinetic energy), and that the negative values reflect only where zero was chosen, since only differences in U_G matter (no credit for this part without the correct selection) — **1 point**

Trap: "Inconsistent, because potential energy can't be negative." The zero of U_G is a choice.

(c) — **1 point**

For selecting "Inconsistent" and correctly explaining that at greater speed an object covers more distance in the same time, so the greater-speed line should be the steeper one; the labels contradict the reasoning in (a)(iii) that moving faster through a slot takes less time (no credit for this part without the correct selection) — **1 point**

(d) — **2 points**

For identifying an incorrect feature: the x-acceleration is not $-g$ (gravity is vertical, and on the curved toy the acceleration is not constant), or the vertical height change $h - h_{\mathrm{PL}}$ is not an x-displacement — **1 point**

For a correct justification, for example: the equation applies only to constant acceleration along the same axis as the displacement; here the horizontal displacement across the slot is x_1 and the horizontal acceleration is neither $-g$ nor constant. (Energy conservation gives the speed at x_1; the x-velocity there equals that speed only because Toy B is level at its right edge.) — **1 point**

Either incorrect feature, with a correct justification, earns both points.

(e) **2 points**

For reasoning that the climbs up A take the same time in both assignments, so the difference is between crossing a level slot of width x_1 at v_0 (on D) and at the speed v_H the ball has at height H (on E): $\Delta t = \dfrac{x_1}{v_H} - \dfrac{x_1}{v_0}$ **1 point**

For using energy conservation to find the speed on E, $v_H = \sqrt{v_0^2 - 2g(H - h_{\mathrm{PL}})}$, giving $\Delta t = x_1/\sqrt{v_0^2 - 2g(H - h_{\mathrm{PL}})} - x_1/v_0$ **1 point**

Score (e) independently of (a)(iii); a difference written in the opposite order (a negative result) earns full credit if both times are correct.

(f) **2 points**

For applying conservation of mechanical energy to the ball–Earth system between the platform and $x = x_1$ (the normal force does no work on the frictionless ball), with the height drop $h_{\mathrm{PL}} - h$: $\frac{1}{2}mv_0^2 + mgh_{\mathrm{PL}} = \frac{1}{2}mv^2 + mgh$ (or the work–energy theorem for the ball alone, with the work done by gravity $mg(h_{\mathrm{PL}} - h)$ and no work done by the normal force) **1 point**

For $v = \sqrt{v_0^2 + 2g(h_{\mathrm{PL}} - h)}$ **1 point**

Trap: Reusing the child's equation from (d). It happens to give the same expression, because the speed gained depends only on the height drop, but a constant-acceleration equation does not describe motion along the curved toy. Such a response earns the second point only.

Question 7.3 (Ramp, collision, stuck blocks, page 46) 7 points

(a) **1 point**

For selecting $U_{G,\mathrm{A\&E}} > U_{G,\mathrm{B\&E}}$ and correctly explaining with $U_G = mgh$ and the chosen zero: block A is at height H, so $U_{G,\mathrm{A\&E}} = m_A gH > 0$, while block B is on the floor, so $U_{G,\mathrm{B\&E}} = 0$, whatever the masses (no credit for this part without the correct selection) **1 point**

(b) **3 points**

For using energy conservation on the frictionless ramp: block A reaches the floor with speed $\sqrt{2gH}$ **1 point**

For using conservation of momentum (not of kinetic energy) for the sticking collision: $m_A\sqrt{2gH} = (m_A + m_B)\, v_{\mathrm{A\&B}}$ **1 point**

For $K = \dfrac{1}{2}(m_A + m_B)\, v_{\mathrm{A\&B}}^2 = \dfrac{m_A^2\, gH}{m_A + m_B}$ **1 point**

A response that conserves mechanical energy through the collision ($K = m_A gH$) earns only the first point: the collision converts some kinetic energy to internal energy.

(c) **1 point**

For selecting $K_2 > K_1$ and correctly explaining that block A's speed at the floor, $\sqrt{2gH}$, does not depend on m_A, so with a larger m_A the stuck blocks have both a larger common speed, $\dfrac{m_A\sqrt{2gH}}{m_A + m_B}$, and a larger total mass (or that $\dfrac{m_A^2\, gH}{m_A + m_B} = \dfrac{m_A\, gH}{1 + m_B/m_A}$ increases with m_A) (no credit for this part without the correct selection) **1 point**

Score (c) for consistency with the expression found in (b), provided that expression is not $K = m_A gH$.

(d) **2 points**

For using energy conservation for the collision: the energy converted is block A's kinetic energy just before the collision, $m_A gH$, minus the kinetic energy found in (b) **1 point**

For $\Delta E_{\mathrm{int}} = m_A gH - \dfrac{m_A^2\, gH}{m_A + m_B} = \dfrac{m_A m_B\, gH}{m_A + m_B}$ **1 point**

Question 7.4 (Block on a free wedge, page 48) — 7 points

(a) — **1 point**

For selecting "Greater than" and correctly explaining that the horizontal momentum of the block–wedge system is conserved and at maximum height the block and wedge move together, so their common speed is the block's initial momentum divided by the total mass, $\frac{(\text{block's mass})\,v}{m+M}$; the total mass is the same in both experiments, and the block's initial momentum is larger in the second ($Mv > mv$) (no credit for this part without the correct selection) — **1 point**

(b) — **6 points**

Correct selection: "Less than." No point is awarded for the selection itself. $H = \frac{M}{m+M} \cdot \frac{v^2}{2g}$ *in the first experiment and* $\frac{m}{m+M} \cdot \frac{v^2}{2g}$ *in the second.*

For reasoning that the horizontal momentum of the block–wedge system is conserved (all surfaces frictionless; the external forces are vertical) — **1 point**

For reasoning that at maximum height the block is at rest relative to the wedge, so both move with the same horizontal velocity, $V = \frac{(\text{block's mass})\,v}{m+M}$ — **1 point**

For using conservation of mechanical energy for the block–wedge–Earth system (no friction): the initial kinetic energy equals the kinetic energy of the common motion plus the gravitational potential energy at maximum height — **1 point**

For finding (or arguing) that the energy left for height is the initial kinetic energy times the wedge's share of the total mass, $\frac{m_{\text{wedge}}}{m+M}$, so $gH = \frac{m_{\text{wedge}}}{m+M} \cdot \frac{v^2}{2}$ — **1 point**

For concluding that swapping the masses makes the wedge's share smaller ($m < M$), so the new height is less than H — **1 point**

For a response that is logical, follows from start to finish, and contains no incorrect or irrelevant statements — **1 point**

Trap: "Greater: the heavier block has more kinetic energy to turn into height." More of that energy also stays as kinetic energy of the shared motion, and the height depends on energy per unit mass of the block.

Question 7.5 (Spring launcher, page 49) — 7 points

(a) — **1 point**

For selecting $E_{\text{MECH,C}} = E_{\text{MECH,B}}$ and correctly explaining that the elastic potential energy belongs to the spring (or the ball–spring system), not to the ball, so the ball's mechanical energy is only its kinetic energy, which is zero in both experiments because the ball is at rest (if the gravitational potential energy of the ball–Earth system is included, it is also the same, because the launcher is horizontal) (no credit for this part without the correct selection) — **1 point**

Trap: "$E_{\text{MECH,C}}$ is greater, because the spring is compressed more." The question asks about the ball, not the ball–spring system.

(b) — **4 points**

Correct selection: $v_{HOR} > v_{VER}$. *No point is awarded for the selection itself.*

For reasoning that the spring releases the same elastic potential energy in both trials (from B to the relaxed position A), or equivalently does the same positive work on the ball — **1 point**

For reasoning that in the vertical trial the ball rises the distance from B to A while it is being launched, so part of that energy becomes gravitational potential energy of the ball–Earth system, while in the horizontal trial none does (or: in the vertical trial the gravitational force does negative work $-mgd$ on the ball from B to A, and in the horizontal trial it does none) — **1 point**

For concluding from this that less energy is left as kinetic energy of the ball in the vertical trial (or that the net work on the ball is smaller), so its launch speed is smaller — **1 point**

For a response that is logical, follows from start to finish, and contains no incorrect or irrelevant statements **1 point**

(c) **2 points**

For energy conservation for the ball–spring–Earth system from release at B to position A, including the gravitational term: $\frac{1}{2}kd^2 = \frac{1}{2}mv_{\text{VER}}^2 + mgd$ **1 point**

For $v_{\text{VER}} = 3.0$ m/s ($\frac{1}{2}kd^2 = 0.25$ J, $mgd = 0.0245$ J with $g = 9.8$ m/s^2, or 0.025 J with $g = 10$ m/s^2) **1 point**

Trap: Omitting mgd gives 3.2 m/s, which is v_{HOR}.

Chapter 8a: Rotational statics

Question 8a.1 (Rod in a bowl, page 51) 7 points

Correct selection: P is on line ℓ. The selection earns no credit by itself; the points come from the paragraph.

For stating that in static equilibrium the net torque on the rod is zero about every axis, so any axis may be chosen (a response that simply states that the net torque about P is zero because the rod is in equilibrium also earns this point) **1 point**

For choosing an axis through point P (or, equivalently, arguing about torques about P) **1 point**

For reasoning that both normal forces have lines of action through P, so each exerts zero torque about P (zero lever arm) **1 point**

For concluding that the gravitational force must then also exert zero torque about P **1 point**

For reasoning that a nonzero force exerts zero torque about P only if its lever arm about P is zero, that is, only if its line of action passes through P **1 point**

For concluding from this reasoning that P lies on line ℓ, the line of action of the gravitational force (checking the blank does not earn this point) **1 point**

For a response that is logical, follows from start to finish, and contains no incorrect or irrelevant statements **1 point**

Example response: When the rod is in static equilibrium, the torques on it add to zero about any axis I choose, so I choose an axis through P. Both normal forces point along lines that pass through P, so their lever arms about P are zero and they exert no torque about P. That leaves the gravitational force, whose torque about P must therefore also be zero. Gravity is not zero, so its lever arm about P must be zero, which means its line of action, line ℓ, passes through P.

An alternative valid argument: three non-parallel forces on a body in equilibrium must have lines of action that meet at one point. It earns the points if it is justified by zero net torque, not merely asserted.

Trap: Keeping the figure's arrangement ("to the left, as shown") without noticing that the figure was said not to show equilibrium, or choosing the axis at the bowl's center or at the tip out of habit and getting lost in lever arms.

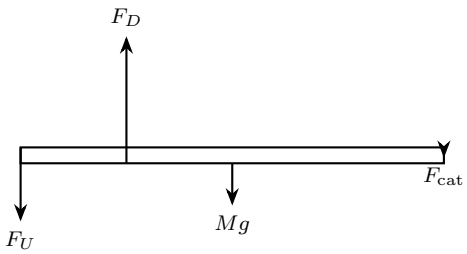
Question 8a.2 (Shelf on two pegs, page 52) — 12 points

(a) — **3 points**

For a downward force by the upper peg at the wall end and an upward force by the lower peg at $L/4$ — **1 point**

For the board's weight drawn downward at $L/2$ and the cat's force drawn downward at L — **1 point**

For no extra forces (no force from the wall, no horizontal forces), all forces labeled distinctly — **1 point**



Courtesy figure, drawn for $m = M/4$ so that $F_D = F_U + Mg + mg$ (arrow lengths are not graded in (a)). Points attach to the features described in words.

Trap: Drawing both peg forces upward, as if the board sat on two supports. With the cat at the far end, the board would rotate down at the cat's end and up at the wall end; the upper peg must hold the wall end down.

(b)(i) — **1 point**

For selecting the ranking $F_D > F_U > Mg$ and correctly explaining both comparisons: the vertical force balance, $F_D = F_U + Mg + mg$, makes F_D the largest; and a torque comparison makes $F_U > Mg$ (for example, about the lower peg, the upper peg's force and the board's weight act at the same distance $\frac{L}{4}$ on opposite sides, and the upper peg must also balance the cat's torque) (no credit for this part without the correct ranking) — **1 point**

Any correct combination of torque and force balances, about any axis, is acceptable.

(b)(ii) — **2 points**

For a torque balance with correct lever arms and directions about a chosen axis (with the force balance if needed to eliminate F_D); for example, about the lower peg: $F_U \cdot \frac{L}{4} = Mg \cdot \frac{L}{4} + mg \cdot \frac{3L}{4}$ — **1 point**

For $F_U = (M + 3m)g$ — **1 point**

(c) — **3 points**

For setting the net torque to zero about a chosen axis, with correct lever arms and directions; for example, about the upper peg, where the lower peg's torque opposes the two weights' torques: $F_D \cdot \frac{L}{4} = Mg \cdot \frac{L}{2} + mg \cdot L$ — **1 point**

For dealing correctly with the unknown F_U: about the upper peg, F_U has zero lever arm and drops out, so the torque equation alone gives F_D (stated or clearly used); about any other axis, combining the torque equation with the vertical force balance $F_D = F_U + Mg + mg$ to eliminate F_U — **1 point**

For $F_D = (2M + 4m)g$ — **1 point**

Any axis earns full credit if the method is carried through. The force-balance route also earns both of the first two points: $F_D = F_U + Mg + mg$ with $F_U = (M + 3m)g$ from (b)(ii) (each step stated). The answer does not contain L.

(d) — **1 point**

For selecting "Decrease" and correctly explaining, for example with torques about the lower peg, that the cat's torque, which the upper peg must balance, gets smaller as the cat's lever arm shrinks from $\frac{3L}{4}$ to $\frac{L}{4}$, while the upper peg's lever arm ($\frac{L}{4}$) is unchanged, so F_U must decrease (to $Mg+mg$); torques about the wall end combined with the force balance are equally valid (no credit for this part without the correct selection) — **1 point**

Trap: "Stay the same: the cat's weight didn't change." Each peg's force depends on where the weight sits, not only on how much there is.

(e) — **2 points**

For a torque balance about the lower peg (or an equivalent combination of torque and force balances) with the cat's torque now on the same side of the lower peg as the upper peg's, both opposing the board's: $F_U \cdot \frac{L}{4} + mg \cdot \frac{L}{4} = Mg \cdot \frac{L}{4}$ **1 point**

For $F_U = (M - m)g$ **1 point**

Trap: Keeping the cat's torque on the board's side and writing $F_U = (M + m)g$. With the cat at the wall end, beside the upper peg, the cat's weight helps hold the wall end down.

Chapter 8b: Rotational dynamics and angular momentum

Question 8b.1 (Five disks, page 54) — 7 points

(a) **4 points**

For correctly comparing torques using $\tau = r_\perp F$: A, B, C, and D all have torque FR (in B, twice the force acts at half the distance); in E the force's line of action passes through the axle, so the torque is zero **1 point**

For correctly comparing rotational inertias: the clay adds rotational inertia, and clay farther from the axle adds more because each lump contributes mr^2; so $I_\mathrm{C} > I_\mathrm{D} > I_\mathrm{A} = I_\mathrm{B}$ **1 point**

For combining these with Newton's second law for rotation, $\alpha = \frac{\Sigma\tau}{I}$, to conclude that A = B > D > C > E: equal torques with the smallest inertia give the largest angular acceleration, and E has none **1 point**

For a logical, relevant paragraph that connects these ideas to the ranking without contradictory or incorrect statements **1 point**

Example response: The torque from each force is the force times its lever arm. A, C, and D each have F acting at R, and B has $2F$ acting at $\frac{R}{2}$, so all four have torque FR. The force on E points straight away from the axle, so its lever arm is zero and so is its torque. The clay adds rotational inertia, and each lump adds its mass times the square of its distance from the axle, so the clay in C adds more than the clay in D; A and B have only the bare disk's inertia. By $\alpha = \frac{\Sigma\tau}{I}$, the same torque gives the largest angular acceleration where the inertia is smallest: A = B > D > C, and E, with no torque, is last.

The ranking blanks are not scored. Values for reference: $\alpha_A = \alpha_B = \frac{2F}{MR}$, $\alpha_D = \frac{8F}{5MR}$, $\alpha_C = \frac{F}{MR}$, $\alpha_E = 0$. *The reasoning points do not require these values.*

Trap: Ranking B above A because $2F > F$; ranking E with A because the force has the same size. Both come from reading the force arrow's length instead of the torque.

(b) **3 points**

For the rotational inertia of disk D including the clay: $I_D = \frac{1}{2}MR^2 + 2 \cdot \frac{M}{4}\left(\frac{R}{2}\right)^2 = \frac{5}{8}MR^2$ **1 point**

For using Newton's second law for rotation with torque FR and constant angular acceleration from rest, $\omega = \alpha\,\Delta t = \frac{FR}{I_D}\Delta t$ (or the angular impulse–angular momentum theorem, $FR\,\Delta t = I_D\,\omega$) **1 point**

For the correct expression $\omega = \dfrac{8F\,\Delta t}{5MR}$ **1 point**

Trap: Leaving out the clay ($I = \frac{1}{2}MR^2$) gives $\frac{2F\Delta t}{MR}$; earns 1 of 3. Placing the lumps at R instead of $\frac{R}{2}$ gives $I = MR^2$; earns 1 of 3.

Question 8b.2 (Pull-cord top, page 55) — 12 points

(a)(i) **2 points**

For explaining in terms of force that the work is the same in both trials: the string pulls with the same force F along the direction its end moves, and its end moves the same distance L (the full length of string) in each trial **1 point**

For explaining in terms of torque that the work is the same: with the bigger sleeve the torque is twice as large, but the same length of string wraps around twice the circumference, so the top turns through half the angle, and the work done by the torque, which depends on torque times angle turned, is unchanged **1 point**

A response that states only that "the torque is larger but acts through a smaller angle," without showing that the two effects cancel, does not earn the second point.

Example response: The string pulls with the same force and its end moves the same distance, the whole length L, in both trials, so it does the same work. In terms of torque: the bigger sleeve doubles the torque, but the string wraps around a circumference twice as long, so the same string unwinds in half as many turns. Twice the torque through half the angle does the same work.

(a)(ii) **1 point**

For selecting "The same as" and correctly explaining that with no friction all of the work done by the string becomes rotational kinetic energy of the top, so the top has the same rotational kinetic energy in both trials, and with the same rotational inertia the same kinetic energy means the same angular speed (no credit for this part without the correct selection) **1 point**

An angular-impulse explanation also earns the point if it shows the cancellation: the torque doubles and the time of pulling halves, so the angular impulse and therefore the final angular momentum are unchanged, and with the same rotational inertia the angular speed is the same.

Trap: "Greater than: the larger sleeve gives more torque." The torque is larger, but it acts through a smaller angle.

(b) **3 points**

For a correct starting relationship with its physical basis identified: with no friction, the work done by the string on the top equals the rotational kinetic energy the top gains, $W = \frac{1}{2}I\omega^2$; or Newton's second law for rotation, $\alpha = \frac{Fr}{I}$, with rotational kinematics for constant angular acceleration from rest, $\omega^2 = 2\alpha\,\Delta\theta$ **1 point**

For justifying the connection between the string length and the motion: the work is FL because the force acts along the string while its end moves a distance L (or because the sleeve turns through $\Delta\theta = \frac{L}{r}$, so $W = \tau\,\Delta\theta = Fr \cdot \frac{L}{r}$); in the kinematics route, $\Delta\theta = \frac{L}{r}$ **1 point**

For $\omega = \sqrt{\dfrac{2FL}{I}}$ **1 point**

No particular method is required. The answer contains no r; a final answer containing r earns at most 2 points. A bare $FL = \frac{1}{2}I\omega^2$, with neither the work–energy identification nor a reason the work is FL, earns the third point only.

(c)(i) **2 points**

For identifying as correct that the torque doubles, with the reason that $\tau = rF$ with the same force F acting perpendicular to a lever arm twice as long **1 point**

For identifying as correct that the angular acceleration doubles, with the reason that $\alpha = \frac{\tau}{I}$ and the rotational inertia is unchanged because the sleeve has negligible mass **1 point**

(c)(ii) **2 points**

For identifying as incorrect the conclusion that the top ends up spinning faster, with the reason that the student ignores that the same length of string wraps around a sleeve with twice the circumference, so it unwinds through half the angle (and in less time), and the doubled angular acceleration acts for less; or for naming the omission, that the student assumes the larger torque acts through the same angle (or for the same time) **1 point**

For explaining that the two changes cancel exactly, so the final angular speed is the same: twice the angular acceleration through half the angle gives the same $\omega^2 = 2\alpha\,\Delta\theta$; or twice the torque through half the angle does the same work and gives the same rotational kinetic energy **1 point**

A response to (c)(ii) that says only "the conclusion is wrong" earns nothing. The second point requires showing that the effects cancel, not only that they act in opposite directions.

(d) **2 points**

For identifying a step in the derivation in (b) that does not depend on the sleeve radius: work $W = FL$ (contains no r); or the product $\alpha\,\Delta\theta = \frac{Fr}{I} \cdot \frac{L}{r}$ in which r cancels **1 point**

For explaining that because this step does not depend on the sleeve radius, doubling the torque cannot change the final angular speed, so the student's conclusion that the top spins faster is incorrect **1 point**

Score (d) against the method used in (b). An energy derivation and a kinematics derivation each have a valid step to point to.

Question 8b.3 (Stacked disks, page 57) — 7 points

(a) — **1 point**

For selecting $\omega_{f,2} = \omega_{f,1}$ and correctly explaining that the two-disk system's angular momentum about the axis is conserved (no external torque about the axis), so $\omega_f = \dfrac{I_{\text{top}}}{I_{\text{top}} + I_{\text{bottom}}}\omega$, and scaling both masses scales both rotational inertias by the same factor, leaving the ratio unchanged (no credit for this part without the correct selection) — **1 point**

The ratio is $\frac{1}{3}$: $\omega_f = \omega/3$ in both experiments.

(b) — **4 points**

Correct conclusion: the same length. No point is awarded for stating the conclusion alone; it is earned with the third point.

For reasoning that the friction forces at the feet are proportional to the normal force, which is proportional to the upper disk's weight, so the frictional torques on both disks grow by the same factor as the masses — **1 point**

For reasoning that the rotational inertias grow by the same factor, so each disk's angular acceleration (τ/I) is unchanged — **1 point**

An energy route earns the first three points: the friction force and the kinetic energy that must be dissipated (the initial minus the final kinetic energy) both scale with the masses, so the sliding distance, friction work divided by friction force, is unchanged.

For concluding that each disk's angular velocity changes in exactly the same way over time, so the time until they match and the angle by which the upper disk slips relative to the lower disk are unchanged; the arc length is the feet's distance from the axis times that relative angle, so the arcs are the same length — **1 point**

For a response that is logical, follows from start to finish, and contains no incorrect or irrelevant statements — **1 point**

Trap: "Longer, because the heavier disk presses harder and has more angular momentum to lose," or "shorter, because the larger friction stops the slipping sooner." Each answer counts only one of two effects that cancel.

(c) — **2 points**

For finding the final angular speed from conservation of angular momentum, $\frac{1}{2}MR^2\omega = \left(\frac{1}{2}MR^2 + \frac{1}{2}(2M)R^2\right)\omega_f$, so $\omega_f = \dfrac{\omega}{3}$, and using it in the final rotational kinetic energy, $\dfrac{1}{2}\left(\dfrac{3}{2}MR^2\right)\left(\dfrac{\omega}{3}\right)^2 = \dfrac{1}{12}MR^2\omega^2$ — **1 point**

For the energy converted as the initial minus the final kinetic energy: $\dfrac{1}{4}MR^2\omega^2 - \dfrac{1}{12}MR^2\omega^2 = \dfrac{1}{6}MR^2\omega^2$ — **1 point**

Trap: Assuming kinetic energy is conserved, or that the final angular speed is $\omega/2$ (equal masses would give that; the lower disk is twice as massive).

Question 8b.4 (Lazy Susan, page 59) — 12 points

(a) — **1 point**

For selecting the ball at the beginning of the first experiment and correctly explaining that both have angular momentum of magnitude $L_1 = I\omega$, so the object with the smaller rotational inertia, the ball ($I_{\text{BALL}} < I_{\text{DISK}}$), has the larger angular speed (no credit for this part without the correct selection) — **1 point**

(b) — **1 point**

For explaining that the ramp's surface rises in the ball's direction of motion, so the normal force from the ramp on the ball has a component opposite the ball's motion (slowing it); by Newton's third law the ball pushes on the ramp with a component in the ball's direction of motion, tangent to the circle, which exerts a counterclockwise torque on the disk about the axle — **1 point**

(c) **2 points**

For reasoning that the angular momentum of the ball–disk system about the axle is conserved: the axle is frictionless, the forces between ball and ramp are internal, and gravity, parallel to the axle, exerts no torque about it; so the total angular momentum is L_1 in both experiments **1 point**

For reasoning that at maximum height the ball is at rest relative to the ramp, so ball and disk rotate together with the same combined rotational inertia in both experiments; with the same angular momentum they have the same angular speed, so the kinetic energy at maximum height is the same in both experiments, $\dfrac{L_1^2}{2(I_{\text{BALL}} + I_{\text{DISK}})}$ (a bare statement that the energies are equal earns neither point) **1 point**

(d) **1 point**

For selecting "Greater than" and correctly explaining that in each experiment only one object is initially moving, with angular momentum L_1, so its kinetic energy is $\dfrac{L_1^2}{2I}$ (or $\frac{1}{2}L_1\omega$ with $\omega = L_1/I$); for the same L_1, the smaller rotational inertia, the ball's ($I_{\text{BALL}} < I_{\text{DISK}}$), gives the larger kinetic energy (no credit for this part without the correct selection) **1 point**

Trap: "Equal, because the angular momentum is the same." Equal angular momenta do not mean equal kinetic energies: $K = L^2/2I$.

(e) **1 point**

For selecting $h_1 > h_2$ and correctly explaining that, with no friction, mechanical energy is conserved, so the decrease in kinetic energy from the start to maximum height becomes gravitational potential energy mgh; the final kinetic energy is the same in both experiments and the initial kinetic energy is greater in the first, so the decrease, and therefore the height, is greater in the first (no credit for this part without the correct selection) **1 point**

(f) **4 points**

For conservation of angular momentum: $L_1 = (I_{\text{BALL}} + I_{\text{DISK}})\,\omega_f$ **1 point**

For the initial kinetic energy $\dfrac{L_1^2}{2I_{\text{BALL}}}$ (or $\frac{1}{2}I_{\text{BALL}}\omega_0^2$ with $\omega_0 = L_1/I_{\text{BALL}}$) **1 point**

For conservation of energy: $\dfrac{L_1^2}{2I_{\text{BALL}}} = \dfrac{L_1^2}{2(I_{\text{BALL}} + I_{\text{DISK}})} + mgh_1$ **1 point**

For $h_1 = \dfrac{L_1^2\, I_{\text{DISK}}}{2mg\, I_{\text{BALL}}\,(I_{\text{BALL}} + I_{\text{DISK}})}$ (or an equivalent form) **1 point**

(g) **2 points**

For h_2 by the same method, with the disk initially moving: $\dfrac{L_1^2}{2I_{\text{DISK}}} = \dfrac{L_1^2}{2(I_{\text{BALL}} + I_{\text{DISK}})} + mgh_2$, **1 point**

so $h_2 = \dfrac{L_1^2\, I_{\text{BALL}}}{2mg\, I_{\text{DISK}}\,(I_{\text{BALL}} + I_{\text{DISK}})}$

For $\dfrac{h_1}{h_2} = \left(\dfrac{I_{\text{DISK}}}{I_{\text{BALL}}}\right)^2$ **1 point**

Score (g) for consistency with the student's h_1 from (f). The result is greater than 1, as found in (e).

Chapter 10: Simple harmonic motion

Question 10.1 (Block between two springs, page 62) 7 points

(a) **3 points**

For recognizing that the block is not attached, so it is pushed by only one spring at a time: it spends half of each cycle on the left spring and half on the right spring **1 point**

For the time on the left spring, half of a mass–spring period, $\pi\sqrt{m/k_L}$, with the time on the right spring, $\pi\sqrt{m/k_R}$, negligible because $k_R \gg k_L$ **1 point**

For $T \approx \pi\sqrt{\dfrac{m}{k_L}}$ **1 point**

Trap: Writing $T = 2\pi\sqrt{m/k_L}$ (the attached-spring period) or $2\pi\sqrt{m/(k_L + k_R)}$ (springs attached on both sides). The block is not attached to either spring. Earns at most 1 point.

(b)(i) **2 points**

For a first correct feature with a correct physics explanation **1 point**

For a second correct feature with a correct physics explanation **1 point**

Part (b) no longer assumes which spring is stiffer (the 2021 wording could be read as $k_R > k_L$, which would make the longer positive half look wrong). Identifying a feature without a correct explanation earns no credit. Acceptable correct features, with explanations: (1) The positive and negative parts last different times (1 s and 0.5 s): each part is half a cycle of a different spring, and $\pi\sqrt{m/k}$ depends on k, which differs. (2) Each part has the shape of half a sine curve: while touching one spring, the block undergoes simple harmonic motion. (3) The motion repeats with period 1.5 s, the sum of the two half-periods: nothing dissipates energy. (4) The graph changes sides only at $x = 0$: that is where the block leaves one spring and meets the other.

(b)(ii) **2 points**

For identifying the equal amplitudes on the two sides as incorrect, explained with energy: the block's kinetic energy at $x = 0$ becomes spring energy at each turning point, $\frac{1}{2}k_L A_L^2 = \frac{1}{2}k_R A_R^2$, so the stiffer spring compresses less. (The side with the shorter half-period, the stiffer spring, should have the smaller amplitude.) **1 point**

For identifying the sudden change in slope at each crossing of $x = 0$ (at $t = 1$ s, 1.5 s, 2.5 s) as incorrect, explained by noting that the slope of an x–t graph is the velocity and the block passes through $x = 0$ with the same speed in both directions (no friction, both springs relaxed there) **1 point**

The two incorrect features are connected (each half has maximum speed $A\omega$, so equal speeds with unequal ω require unequal A); a student may identify them in either order. Other genuinely incorrect features with correct explanations also earn credit.

Trap: Calling the unequal half-durations an error ("SHM is symmetric") or calling the equal amplitudes correct ("the amplitude is set by how the block was started"). Only one amplitude is set by the start; the other follows from energy.

Question 10.2 (Skate ramp, page 63) 7 points

(a) **4 points**

Correct selection: "Increase in T." The selection earns no credit by itself; the points come from the paragraph.

For reasoning that on each curved part (with $h \ll R$) the ball moves like a simple pendulum of length R: it spends half a pendulum period, $\pi\sqrt{R/g}$, there, independent of how high it started **1 point**

For reasoning that the time on the flat segment, L divided by the speed $\sqrt{2gh}$ (set by energy conservation), does not depend on R **1 point**

For concluding from these two pieces of reasoning that a larger R lengthens the time on the curves and leaves the flat-segment time unchanged, so T increases: $T = 2\pi\sqrt{R/g} + \dfrac{2L}{\sqrt{2gh}}$ **1 point**

For a response that is logical, follows from start to finish, and contains no incorrect or irrelevant statements **1 point**

Example response: Because h is much smaller than R, the ball on each curved part moves like a simple pendulum of length R, so in each round trip it spends half a pendulum period, $\pi\sqrt{R/g}$, on each curve, however high it was released. On the flat part it moves at the speed it gained by dropping h, $\sqrt{2gh}$, so it crosses in $L/\sqrt{2gh}$, which has nothing to do with R. One round trip includes both curves and two crossings, $T = 2\pi\sqrt{R/g} + 2L/\sqrt{2gh}$. Increasing R makes the curve time longer and leaves the crossing time unchanged, so T increases.

Trap: "No change, because the period of a pendulum doesn't depend on amplitude." True, but it does depend on length, and here R plays the role of the length.

(b) **3 points**

For identifying a consistent feature, with explanation: T increases as R increases, as reasoned in (a) **1 point**

For identifying an inconsistent feature: the best-fit line passes through the origin (or: the trend is a straight line) **1 point**

For explaining it: even as R becomes small, the ball still needs time $\frac{2L}{\sqrt{2gh}}$ to cross the flat segment twice, so T cannot approach zero (or: the curved-part time grows like $\sqrt{R}$, so T vs. R should be a curve that flattens, not a straight line) **1 point**

Question 10.3 (Friction that grows with distance, page 64) 7 points

(a) **4 points**

Correct selection: C. No point is awarded for the selection itself.

For reasoning that the friction force has magnitude $\mu_K mg = bmg\,x$, so by Newton's second law the acceleration has magnitude $bg\,x$, proportional to the distance into the darker region, and points backward **1 point**

For explaining the shape, either by recognizing that a backward force proportional to x acts like a spring force (with effective spring constant bmg), so while the block slides, x follows the first quarter of a sine curve: it increases quickly at first and then more slowly, leveling off as the block comes to rest; or kinematically: $|a| \propto x$, so $|a|$ grows at a rate proportional to the speed, which falls to zero **1 point**

For concluding from this reasoning that $|a|$ starts at zero, rises at a decreasing rate, and levels off at its maximum when the block stops, which is graph C **1 point**

For a response that is logical, follows from start to finish, and contains no incorrect or irrelevant statements **1 point**

Trap: Choosing D ("friction grows steadily") or B ("the block slows down, so the acceleration decreases"). The acceleration grows with distance, not with time, and the block covers distance more slowly as it slows.

(b) **3 points**

For identifying the motion as a quarter cycle of simple harmonic motion (from the equilibrium position to a turning point) **1 point**

For the angular frequency from the effective spring constant: $\omega = \sqrt{\dfrac{bmg}{m}} = \sqrt{bg}$ (or the period $2\pi\sqrt{1/(bg)}$) **1 point**

For $t = \dfrac{\pi}{2}\sqrt{\dfrac{1}{bg}}$, independent of v_0 **1 point**

Trap: Including v_0 in the answer. As for any simple harmonic motion, the time is independent of the amplitude.

Question 10.4 (Pendulum with a lower rod, page 66) 7 points

(a) **4 points**

Correct selection: "Decrease." No point is awarded for the selection itself.

For reasoning that Δt has two parts: swinging down to the lowest point as a pendulum of length ℓ, and swinging up the other side as a pendulum of length $\ell - L$ (the part of the string below rod B) **1 point**

For reasoning that each part is a quarter of the period of a simple pendulum, $2\pi\sqrt{\text{length}/g}$, which for small angles does not depend on how far the bob swings **1 point**

For concluding from this reasoning that increasing L leaves the first part unchanged and shortens the second pendulum, which shortens its quarter period, so Δt decreases **1 point**

For a response that is logical, follows from start to finish, and contains no incorrect or irrelevant statements **1 point**

Trap: "Increase, because a larger L makes the apparatus longer," or "stay the same, because the string length did not change." Only the part of the string that swings counts.

(b) **3 points**

For the first quarter period, $\frac{\pi}{2}\sqrt{\frac{\ell}{g}}$ **1 point**

For the second quarter period, $\frac{\pi}{2}\sqrt{\frac{\ell - L}{g}}$ **1 point**

For $\Delta t = \frac{\pi}{2}\left(\sqrt{\frac{\ell}{g}} + \sqrt{\frac{\ell - L}{g}}\right)$ **1 point**

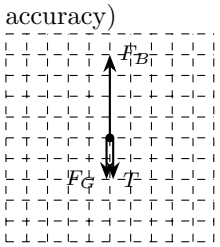
Chapter 11a: Fluid statics

Question 11a.1 (Float in a fish tank, page 68) **7 points**

(a) **2 points**

For three forces and only three, each labeled distinctly: upward buoyant force, downward gravitational force, downward tension **1 point**

For the buoyant arrow longer than each downward arrow (equal to their sum, within drawing accuracy) **1 point**



Courtesy figure. The relative lengths of F_G and T are not graded; they depend on ρ_f.

(b) **3 points**

For treating the tank, water, and float as one system whose only external forces are gravity and the scale's upward force, so the scale's force differs from the total weight only if the system's center of mass accelerates (before the cut nothing accelerates, so the scale reads the total weight) **1 point**

For reasoning that as the float starts to rise, an equal volume of water moves down to take its place; that water has more mass than the float, so the center of mass starts to accelerate downward, and therefore the scale's force, and its reading, is less than before the cut **1 point**

For a logical, relevant paragraph that connects these ideas to the conclusion without contradictory or incorrect statements **1 point**

Example response: Before the cut everything is at rest, so the scale reads the total weight. After the cut the float speeds up upward, but an equal volume of heavier water speeds up downward to fill in. The system's center of mass accelerates downward, so the scale pushes up with less than the total weight.

A correct argument using forces on the tank alone (the string no longer pulls up on the tank bottom, but the water's push on the bottom decreases by more than the string's former pull) also earns the first two points if each step is justified. The answer blanks are not scored.

Trap: "The same as: nothing was added to or removed from the tank." That is true just before the cut, when nothing accelerates; the part asks about just after.

(c) **2 points**

For Newton's second law with zero acceleration and the buoyant force $\rho_w V g$: $\rho_w V g - \rho_f V g - T = 0$ **1 point**

For $T = (\rho_w - \rho_f) V g$ **1 point**

Question 11a.2 (Floating cans, page 69) **7 points**

(a) **6 points**

Correct selection: $d_1 < d_2$. The selection earns no credit by itself; the points come from the paragraph.

For reasoning that each can is in equilibrium before and after, so the buoyant force must increase by exactly the added weight, $\Delta m\, g$, which is the same for both cans **1 point**

For stating that the buoyant force equals the weight of the displaced water, $F_B = \rho_{\text{water}} V_{\text{disp}}\, g$ **1 point**

For concluding that each can must displace the same additional volume of water, $\Delta V = \Delta m/\rho_{\text{water}}$ **1 point**

For relating that volume to the extra depth through the cross-sectional area of the can: $\Delta V = A\, d$ **1 point**

Equivalent route for the second through fourth points: sinking an extra depth d raises the water pressure on the can's bottom by $\rho_{water} g d$, which adds an upward force $\rho_{water} g d\, A$ on the bottom; this must equal $\Delta m\, g$, so $d = \Delta m/(\rho_{water} A)$. A response that argues this way, with each step stated, earns those three points.

For concluding that the can with the larger bottom area (can 1) sinks the smaller extra depth **1 point**

For a response that is logical, follows from start to finish, and contains no incorrect or irrelevant statements **1 point**

Example response: Each can floats both before and after, so the upward buoyant force on it balances its weight. Adding Δm adds weight $\Delta m\, g$, so the buoyant force on each can must grow by that same amount. The buoyant force is the weight of the water the can pushes aside, so each can must push aside the same extra volume of water, $\Delta m/\rho_{\text{water}}$. A can that sinks a little deeper pushes aside an extra slab of water whose volume is its bottom area times the extra depth. Can 1 has the larger bottom, so it needs a thinner slab: $d_1 < d_2$.

Trap: "Equal, because the same mass was added to both." The same added mass gives the same added displaced *volume*, not the same added depth.

(b) **1 point**

For selecting "Less than" and correctly explaining that the floating can's weight is unchanged, so the buoyant force must be the same, and that a denser liquid supplies that buoyant force with a smaller displaced volume ($\rho V_{\text{disp}}\, g$ fixed), so the can sits less deep (no credit for this part without the correct selection) **1 point**

Trap: "Equal: the same can with the same load needs the same buoyant force." True, but the same buoyant force in a denser liquid takes less displaced volume.

Chapter 11b: Fluid flow

Question 11b.1 (Faucet stream, page 70) — 7 points

(a) — **6 points**

Correct selection: "Less than." The selection earns no credit by itself; the points come from the paragraph.

For reasoning that the flow is steady, the water is incompressible, and no water leaves or joins the stream between the points, so the same volume of water passes each point per second — **1 point**

For relating volume flow rate to area and speed: $Q = Av$ (continuity, $A_1 v_1 = A_2 v_2$) — **1 point**

For reasoning that the water speeds up as it falls, because gravity acts on it (the pressure in the open stream is atmospheric throughout, so pressure does not slow it) — **1 point**

This point can also be earned with Bernoulli's equation between points 1 and 2, if the response says that the pressure terms are equal (both atmospheric) and that the decrease in the $\rho g y$ term must therefore appear as an increase in the $\frac{1}{2}\rho v^2$ term. Writing the equation without that narration does not earn the point: the response must leave no reasonable doubt that the student knows why the water speeds up.

For noting that point 2 is lower, so the water is moving faster there: $v_2 > v_1$ — **1 point**

For concluding that a larger speed with the same flow rate requires a smaller area — **1 point**

For a response that is logical, follows from start to finish, and contains no incorrect or irrelevant statements — **1 point**

Example response: The stream is steady and water doesn't compress, and none is added or lost between the two points, so the same volume of water passes point 1 and point 2 each second. That volume per second equals the stream's cross-sectional area times the water's speed. The stream is open to the air all along, so the only thing changing the water's speed is gravity, which speeds it up as it falls; the water is moving faster at point 2. Carrying the same volume per second at a higher speed takes a smaller area, so the stream is thinner at point 2.

Trap: Explaining the narrowing as the air pressure "squeezing" the stream, or as surface tension. Neither is needed, and a response built on either loses the reasoning points it replaces.

(b) — **1 point**

For selecting "Greater than" and correctly explaining that falling the same distance adds the same amount to the square of the speed ($v_2^2 = v_1^2 + 2gh$, where h is the distance from point 1 down to point 2, from energy conservation or Bernoulli's equation), which multiplies a larger v_1 by a smaller factor, so v_2/v_1 is smaller; by continuity $A_2/A_1 = v_1/v_2$, so the ratio is larger (closer to 1); or, equivalently, a kinematic explanation: faster water spends less time falling between the points, so it gains less speed ($\Delta v = g\,\Delta t$), and that smaller gain is a smaller fraction of the larger v_1, so v_2/v_1 is smaller and $A_2/A_1 = v_1/v_2$ is larger (no credit for this part without the correct selection) — **1 point**

A response that says only that the water speeds up by the same amount, Δv, does not earn the point: the gain in speed is not the same; the gain in v^2 is.

Trap: "Equal: the stream falls the same distance, so it narrows the same way." The same drop gives the same increase in v^2, which is a smaller fraction of a larger v_1^2.

Question 11b.2 (Syringe pump and needles, page 71) — 12 points

(a) — **2 points**

For using continuity: $Q = Av$ with $A = \dfrac{\pi d^2}{4}$ — **1 point**

For $v = \dfrac{4Q}{\pi d^2}$ — **1 point**

(b) — **2 points**

For choosing v and $1/d^2$ (in either order), or another pair that gives a straight line from which Q can be found — **1 point**

For a correct column of values, for example $1/d^2 = 2.78, 2.04, 1.42, 0.873, 0.533\ \text{mm}^{-2}$ — **1 point**

(c) — **3 points**

For correctly scaled and labeled axes (with units) that use more than half the grid — **1 point**

For correctly plotted points — **1 point**

For a reasonable best-fit line (through or near the origin, with points on both sides) **1 point**

(d) **2 points**

For using the slope of the best-fit line, in any units (not two data points far from the line); for v against $1/d^2$, about 1.9 m/s per $\text{mm}^{-2} = 1.9 \times 10^{-6}\ \text{m}^3/\text{s}$ **1 point**

For $Q = \dfrac{\pi}{4} \times \text{slope} \approx 1.5 \times 10^{-6}\ \text{m}^3/\text{s}$ (1.5 mL/s); accept 1.4–$1.6 \times 10^{-6}\ \text{m}^3/\text{s}$; or the equivalent for the student's axes (for example, $Q = \dfrac{\pi}{4 \times \text{slope}}$ for $1/d^2$ against v) **1 point**

A student who graphed v against $1/d^2$ with d in mm must convert units; an answer off by 10^6 because of units earns the first point only.

(e) **3 points**

For using a pair of data points with diameters in a ratio of about 2 to show that the statement is incorrect: for example, from $d = 1.37$ mm to $d = 0.70$ mm (about half), the speed rises from 1.03 to 3.82 m/s, about 3.7 times, not 2. (Stating "incorrect" without such a comparison earns no credit.) **1 point**

For showing with the data that the speed is instead inversely proportional to the square of the diameter: for example, vd^2 is nearly the same for every needle (1.85–1.97 $\text{m/s}\cdot\text{mm}^2$), or the ratio 3.7 matches $(1.37/0.70)^2 \approx 3.8$, or the straight line through the origin in (c) **1 point**

For explaining with continuity: the same Q passes through every needle and the area depends on d^2, so halving d quarters the area and quadruples the speed **1 point**

Trap: Agreeing with the student because "speed is inversely proportional to the needle's size." It is inversely proportional to the area.

Chapter 11c: Fluids: statics and flow combined

Question 11c.1 (Aquarium siphon, page 73) **7 points**

(a) **3 points**

For applying Bernoulli's equation along a streamline from the aquarium's surface to the open end of the hose (or energy conservation for a parcel of water moving from the surface to the open end, with the work done by atmospheric pressure at the two ends cancelling) **1 point**

For using atmospheric pressure at both places and negligible speed at the surface of the large aquarium **1 point**

For $v = \sqrt{2gh}$ (no y) **1 point**

Trap: Including y, for example $\sqrt{2g(h+y)}$ or $\sqrt{2g(h-y)}$, because the water "has to climb" first. The climb and the descent of the loop cancel.

(b) **1 point**

For selecting "Stay the same" and correctly explaining that the exit speed depends only on the height difference h between the aquarium's surface and the open end (from (a)), which has not changed; or that the water slows on the way up to the higher top and speeds up again by the same amount on the way down (no credit for this part without the correct selection) **1 point**

(c) **1 point**

For selecting "Less than" and correctly explaining with Bernoulli's equation between the surface and the top of the hose: the water at the top is higher than the surface and moving, so $P_{\text{top}} = P_{\text{atm}} - \rho g y - \frac{1}{2}\rho v^2$, which is below atmospheric (or: the speed in the uniform hose is the same everywhere, so between the top and the open end, where the pressure is atmospheric, the top is higher by $h+y$ and must be at lower pressure) (no credit for this part without the correct selection) **1 point**

(d) **2 points**

For using continuity for the whole system: the volume of water leaving through the hose each second equals the volume by which the aquarium's water decreases each second, $A_\text{s} v_\text{s} = av$ **1 point**

For $v_\text{s} = \dfrac{a}{A_\text{s}}\sqrt{2gh}$ **1 point**

Score the second point for consistency with the student's answer to (a).

Question 11c.2 (Bottle with two holes, page 74) **7 points**

(a)(i) **2 points**

For identifying a correct aspect: the pressure inside the bottle is greater at hole A, or the water leaves hole A faster **1 point**

For a correct reason: pressure increases with depth, $P = P_{\text{atm}} + \rho g d$; or, by Bernoulli's equation, the exit speed is $\sqrt{2gd}$, which is larger for the greater depth d **1 point**

(a)(ii) **2 points**

For identifying the incorrect aspect: the conclusion that the water from hole A lands farther **1 point**

For explaining that the landing distance also depends on how long the water is in the air, which is shorter for the lower hole A, and showing that this effect wins here: with the result of (b), hole A's water lands at $\frac{\sqrt{3}}{2}H \approx 0.87H$ and hole B's at H, so hole B's water lands farther. (Noting the shorter fall time alone shows only that the claim is unsupported, not that it is wrong; it does not earn this point.) **1 point**

A student may compute the distances in (b) and use them here.

Trap: Ranking the landing distances by exit speed alone. Speed and time in the air pull in opposite directions.

(b) **3 points**

For the exit speed from Bernoulli's equation (or energy conservation): $v = \sqrt{2g(H-y)}$ **1 point**

For the time to fall height y from horizontal launch: $t = \sqrt{2y/g}$ **1 point**

For $D = vt = 2\sqrt{y(H-y)}$ **1 point**

AP Physics 2: Scoring Guidelines

Prepared with assistance from Claude, an AI model made by Anthropic. "Example response" entries show one acceptable response, not a required one. **Trap** notes tell the tutor what a careless or pattern-matching reader is likely to write. Numerical answers in these guidelines use $g = 9.8$ m/s^2; answers that use $g = 10$ m/s^2 earn full credit unless the question specifies 9.8.

Chapter 7: Potential-energy landscapes

Question 7.1 (Magnet cart on a track, page 77) — 10 points

(a)(i) — **1 point**

For selecting 7 cm and correctly explaining that the magnitude of the force equals the magnitude of the slope of the U–x graph, which is steepest at 7 cm (between 6 cm and 8 cm the graph rises $4U_0$, the largest change over any of the equal 2 cm intervals) (no credit for this part without the correct selection) — **1 point**

(a)(ii) — **1 point**

For selecting $-x$ and correctly explaining that the graph slopes upward at 3 cm and the force points toward lower potential energy ($F_x = -\Delta U/\Delta x$) (no credit for this part without the correct selection) — **1 point**

Trap: Answering $+x$ because the cart "moves uphill" to the right, or zero because U is not extreme at 3 cm. The force depends on the slope, not on the value of U or on the direction of motion.

(b)(i) — **1 point**

For selecting "No" and correctly explaining with energy conservation that, released from rest at 1 cm, the cart's total energy is $2.5U_0$, which is less than the $3U_0$ peak at 4 cm, so the cart cannot get over the peak (no credit for this part without the correct selection) — **1 point**

(b)(ii) — **2 points**

For reasoning that the total energy stays $2.5U_0$, so the cart can be only where $U \leq 2.5U_0$ (kinetic energy cannot be negative), and the ends of its range are where $U = 2.5U_0$ — **1 point**

For the range from $x = 1$ cm to about $x = 3.3$ cm (accept 3.2–3.4 cm), between 3 cm and 4 cm on the rising part of the curve — **1 point**

Score (b)(ii) for consistency with the total energy the student found in (b)(i).

(c) — **2 points**

For identifying the error: at the minimum of U the kinetic energy is greatest, not zero; the cart moves fastest there — **1 point**

For explaining that the cart is at rest only at turning points, where U equals the total energy; at 2 cm the force is zero, which means the acceleration (not the velocity) is zero — **1 point**

(d) — **3 points**

For using energy conservation with the peak at 4 cm as the limiting point: $\frac{1}{2}mv^2 + U_0 \geq 3U_0$ (once over the peak, the cart reaches 6 cm, where $U = 0$) — **1 point**

For $v_{\min} = 2\sqrt{\dfrac{U_0}{m}}$ (or $\sqrt{4U_0/m}$) — **1 point**

For using energy conservation from 2 cm (total energy just over $3U_0$) to 6 cm, where $U = 0$, to get $\frac{1}{2}mv_6^2 \approx 3U_0$, so $v_6 \approx \sqrt{\dfrac{6U_0}{m}}$ — **1 point**

Trap: Using the energy difference between 2 cm and 6 cm ($1U_0 \to 0$) and concluding that any speed works. The cart has to get over the peak in between.

Question 7.2 (Two bonded atoms, page 79) **12 points**

(a)(i) **1 point**

For $r = r_0$, justified by the force being the negative of the slope of the U graph ($F = -\Delta U/\Delta r$), which is zero at the minimum of U (a bare reading of r_0 does not earn the point) **1 point**

(a)(ii) **1 point**

For selecting "Repulsive" and correctly explaining that at $0.75r_0$ the graph slopes downward as r increases, so the force $F = -\Delta U/\Delta r$ is in the $+r$ direction, pushing the atoms apart (no credit for this part without the correct selection) **1 point**

(b)(i) **1 point**

For about $0.80r_0$ and $1.35r_0$ (accept $\pm 0.05r_0$), where the E_1 line meets the curve **1 point**

(b)(ii) **1 point**

For about $0.69r_0$ (accept $\pm 0.05r_0$) and $2.0r_0$ (accept $\pm 0.1r_0$) **1 point**

(b)(iii) **1 point**

For selecting E_2 and supporting it with averages calculated from the student's estimates in (b)(i) and (b)(ii) (about $1.07r_0$ for E_1 and $1.35r_0$ for E_2) (no credit for this part without the correct selection) **1 point**

(b)(iv) **1 point**

For reasoning that the kinetic energy is greatest where U is least, at $r = r_0$, where $U = -U_0$, so $K_{\max} = E_2 - (-U_0) = 0.75U_0$ **1 point**

(c) **1 point**

For explaining that with a well of the same shape on both sides of its minimum, raising the energy would move the two turning points away from r_0 by equal amounts, so the average of the smallest and largest separations would stay at r_0 at every energy, and heating would not change the average separation **1 point**

(d) **2 points**

For reasoning that heating gives each pair more vibrational energy, which raises the total-energy line on the graph, and that the pair vibrates between the two separations where that line meets the curve (U equals the total energy) **1 point**

For reasoning from the asymmetry: the curve is steep for $r < r_0$ and shallow for $r > r_0$, so raising the energy moves the outer turning point out much more than it moves the inner turning point in (for example, from about $0.80r_0$ and $1.35r_0$ at E_1 to about $0.69r_0$ and $2.0r_0$ at E_2); the average separation therefore increases, neighboring atoms are farther apart on average, and the solid expands **1 point**

Example response: Heating the solid gives each pair of atoms more vibrational energy, which raises the total-energy line on the graph. The pair vibrates between the two separations where that line meets the curve. The curve is very steep at small separations and shallow at large ones, so raising the line pushes the outer turning point much farther out than it pulls the inner one in. The average separation therefore grows, and since every pair is farther apart on average, the solid is larger.

Trap: "The atoms vibrate harder and push each other apart" with no reference to the graph's asymmetry. Bigger vibrations alone don't change the average separation.

(e)(i) **2 points**

For the sign: F positive (repulsive) for $r < r_0$, zero at $r = r_0$, and negative (attractive) for $r > r_0$, approaching zero at large r **1 point**

For the shape: the repulsive force grows steeply as r decreases below r_0 (far larger than any attractive force on the graph), and the attractive force is greatest in magnitude near $r \approx 1.35r_0$, where the U curve is steepest for $r > r_0$, then shrinks toward zero **1 point**

A repulsive branch that rises off the top of the grid is accepted (the repulsive force reaches about $11\,U_0/r_0$ at $0.6r_0$). The force is $F = -\Delta U/\Delta r$. For this model the force is about $+4\,U_0/r_0$ at $0.75r_0$ and has its greatest attractive magnitude, $1.0\,U_0/r_0$, at $1.35r_0$.

(e)(ii) **1 point**

For identifying the steepest part of the curve for $r > r_0$ (near 1.3–1.4 r_0) and using the slope of a tangent there to estimate a magnitude of about $1.0\,U_0/r_0$ (accept 0.8–1.2 U_0/r_0) **1 point**

Trap: Reading the depth of the well, U_0, or the value of U at some separation as the force. The force is the slope of the U graph, not its value.

Chapter 11: Waves and sound

Question 11.1 (Guitar string, page 81) — 12 points

(a) — **3 points**

For the wave speed on a string, $v = \sqrt{T/\mu}$ — **1 point**

For the fundamental wavelength with both ends fixed, $\lambda = 2L$ — **1 point**

For $f = \dfrac{v}{\lambda} = \dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}$ — **1 point**

Trap: Using $\lambda = L$ (one full wave between the ends) gives $f = \frac{1}{L}\sqrt{T/\mu}$; earns 2 of 3.

(b) — **2 points**

For choosing f^2 and T (in either order), or another pair giving a straight line from which μ can be found (for example f and $\sqrt{T}$) — **1 point**

For a correct column of values, for example $f^2 \approx 19\,900$, $29\,200$, $39\,600$, $48\,800$, $59\,500$ Hz^2 — **1 point**

(c) — **3 points**

For correctly scaled and labeled axes (with units) that use more than half the grid — **1 point**

For correctly plotted points — **1 point**

For a reasonable best-fit line — **1 point**

(d) — **2 points**

For using the slope of the best-fit line, about 490–500 Hz^2/N for f^2 vs. T (not two data points far from the line), and relating it to μ: slope $= \dfrac{1}{4L^2\mu}$ — **1 point**

For $\mu \approx 1.2 \times 10^{-3}$ kg/m (accept 1.1–1.3$\times 10^{-3}$ kg/m) — **1 point**

(e) — **2 points**

For about 100 Hz (half of the 199 Hz measured at 80 N) — **1 point**

For reasoning that $f \propto 1/\sqrt{\mu}$ at fixed tension and length, so four times μ halves f — **1 point**

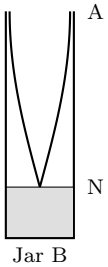
Trap: Predicting one-fourth of the frequency, about 50 Hz, by treating f as inversely proportional to μ.

Question 11.2 (Blowing across jars, page 83) — 10 points

(a)(i) — **2 points**

For a displacement node (N) at the water surface, where the air cannot move, and a displacement antinode (A) at the open top of the jar (or just above the rim) — **1 point**

For a quarter-wavelength pattern: the amplitude grows steadily from the node at the water to the antinode at the top, with no other node or antinode in between, and nothing drawn in the water — **1 point**



Courtesy figure: the curves show the displacement amplitude across the air column, zero at the water (N) and largest at the open top (A). An antinode drawn slightly above the rim is also correct.

(a)(ii) — **1 point**

For selecting "Higher than" and correctly explaining that the vibrating air column is closed at the water and open at the top, so its fundamental wavelength is four times (or at least proportional to) the air column's length; jar A's air column is shorter, so its wavelength is shorter, and with the same speed of sound, $f = v_s/\lambda$ is higher (no credit for this part without the correct selection) **1 point**

Trap: Reasoning from the water ("more water, heavier, lower pitch"). When the jar is blown across, the air column, not the water or the glass, is what vibrates.

(b) **2 points**

For the air-column length $H - d$ and the closed–open fundamental $\lambda = 4(H - d)$ **1 point**

For $f = \dfrac{v_s}{4(H-d)}$ **1 point**

(c)(i) **2 points**

For finding the air-column lengths, $\frac{3H}{4}$ at depth $\frac{H}{4}$ and $\frac{H}{2}$ at depth $\frac{H}{2}$, and using them in the expression from (b) for both depths: $f_1 = \dfrac{v_s}{3H}$, $f = \dfrac{v_s}{2H}$ **1 point**

For $f = \dfrac{3}{2} f_1$ **1 point**

Score (c)(i) for consistency with the student's expression in (b) if it depends on $H - d$.

(c)(ii) **1 point**

For selecting "No" and correctly explaining that the frequency is inversely proportional to the air column's length $H - d$, not proportional to the depth, so doubling the depth from $\frac{H}{4}$ to $\frac{H}{2}$ shortens the column only from $\frac{3H}{4}$ to $\frac{H}{2}$ and multiplies the frequency by $\frac{3}{2}$, not by 2 (or by $\frac{1}{2}$) (doubling it would take a depth of $\frac{5H}{8}$) (no credit for this part without the correct selection) **1 point**

(d) **1 point**

For selecting "Increase" and correctly explaining that the wavelength is fixed by the air column's length, which has not changed, so a greater speed of sound gives a greater frequency, $f = v_s/\lambda$ (no credit for this part without the correct selection) **1 point**

(e) **1 point**

For a ratio of 3, explained by noting that a column closed at one end and open at the other has a node at the closed end and an antinode at the open end, so it holds only odd numbers of quarter wavelengths; the next mode has $L = \frac{3}{4}\lambda$, a wavelength one-third as long, so three times the frequency **1 point**

Trap: Answering 2, as for a string fixed at both ends or a pipe open at both ends. A closed–open column has no even harmonics.

Question 11.3 (Playing music over fiber, page 85) 12 points

(a)(i) **1 point**

For selecting "Less than" and correctly justifying it: total internal reflection can occur only when light strikes a boundary with a medium of smaller index; by Snell's law, $n \sin\theta = n_{\text{cl}} \sin\theta_{\text{cl}}$, so if $n_{\text{cl}} \geq n$ there is always a refracted angle with $\sin\theta_{\text{cl}} \leq 1$ and some light always enters the cladding (no credit for this part without the correct selection) **1 point**

Trap: "Greater than, so the cladding acts like a wall that bounces the light back." Light going into a denser medium always has a refracted ray; total reflection needs the smaller index on the far side.

(a)(ii) **2 points**

For identifying the smallest angle as the critical angle, at which the refracted ray would travel along the boundary (refraction angle 90°): $n \sin\theta_c = n_{\text{cl}} \sin 90°$ **1 point**

For $\theta_c = \sin^{-1}\left(\dfrac{n_{\text{cl}}}{n}\right)$ (or $\sin\theta_c = n_{\text{cl}}/n$) **1 point**

An answer consistent with a reversed choice in (i) does not earn the second point, because $\sin^{-1}(n/n_{cl})$ is undefined for $n < n_{cl}$ unless the student explicitly writes the correct ratio. For typical fibers ($n = 1.47$, $n_{cl} \approx 1.46$), $\theta_c \approx 83°$: only rays traveling nearly along the fiber are guided.

(b) **2 points**

For setting the travel time of sound in the room, d/v_s, equal to the time for the signal to travel the length L in the fiber at speed c/n: $\dfrac{d}{v_s} = \dfrac{L}{c/n} = \dfrac{nL}{c}$ **1 point**

For $d = \dfrac{nv_sL}{c}$ **1 point**

Trap: Using c for the speed in the fiber (dropping n), or dividing by n ($d = v_sL/(nc)$). Light is slower in glass, so the fiber delay is n times the vacuum delay.

(c) **1 point**

For selecting "Less than" and correctly justifying it: with index near 1 the light travels at nearly c, faster than $c/1.47$ in glass, so it travels the same length in less time, and the equivalent separation $d = nv_sL/c$ is smaller, by a factor of about 1/1.47 (no credit for this part without the correct selection) **1 point**

Accept the reasoning stated as "d is proportional to n" from (b), or as consistent with the student's expression in (b) if that expression is wrong.

(d) **2 points**

For explaining that in one room, A also hears B's note only after the sound has traveled from A to B and back, so the delay A notices in the room is also a round-trip time, $2d/v_s$ (both delays double) **1 point**

For concluding that $\dfrac{2d}{v_s} = \dfrac{2nL}{c}$ gives the same separation d as in (b), not $2d$ **1 point**

Example response: The student is right that A waits twice the one-way fiber time, $2nL/c$. But if the musicians stood in one room, A would also wait for the sound to go to B and for B's sound to come back, a time $2d/v_s$. Setting these equal, the factor of 2 cancels, so the separation is still d.

Trap: Agreeing with $2d$, which compares a round trip over the fiber with a one-way trip in the room.

(e) **3 points**

For $t = \dfrac{nL}{c} = \dfrac{(1.47)(1.2 \times 10^6 \text{ m})}{3.00 \times 10^8 \text{ m/s}} \approx 5.9 \times 10^{-3}$ s (5.9 ms) **1 point**

For $d = v_st = (343 \text{ m/s})(5.9 \times 10^{-3} \text{ s}) \approx 2.0$ m (or $d = nv_sL/c$ from (b)) **1 point**

For the length in the fiber $L = \dfrac{c\,d}{nv_s} = \dfrac{(3.00 \times 10^8 \text{ m/s})(8.0 \text{ m})}{(1.47)(343 \text{ m/s})} \approx 4.8 \times 10^6$ m (4,800 km) (or by scaling: $1{,}200 \text{ km} \times \dfrac{8.0 \text{ m}}{2.0 \text{ m}}$) **1 point**

Accept 4,700–4,900 km, or a value consistent with the student's d for 1,200 km when found by scaling. Accept 5.8–6.0 ms and 1.9–2.1 m. Using the straight-line distance, 950 km (about 4.7 ms and 1.6 m), or omitting n (about 4.0 ms and 1.4 m) does not earn the first point but earns the second point if $d = v_st$ is correctly computed from the student's time.

The punchline: over 1,200 km of ideal fiber, the delay matches that of two musicians about 2 m apart in a room, closer than many ensembles sit. In practice the delays the question set aside are usually larger than this.

(f) **1 point**

For selecting "Greater than" and correctly justifying it: the signal travels at least $2 \times 36{,}000 = 72{,}000$ km, about 60 times the 1,200 km in the fiber, while it moves only 1.47 times as fast, so the travel time is about 40 times as long (about 0.24 s, compared with about 6 ms) (no credit for this part without the correct selection) **1 point**

A ratio argument without computing either time earns the point if it compares both the distances and the speeds. The satellite delay matches two musicians about 80 m apart in one room.

Trap: "Less than, because radio waves in space travel at c, faster than light in glass." The speed is only about 1.5 times as great, but the path is about 60 times as long.

Question 11.4 (Two speakers in a theater, page 87) **12 points**

(a) **1 point**

For explaining that R is the same distance from both speakers (it lies on the line midway between them), so the two waves, which leave the speakers in phase, travel equal path lengths and arrive in phase **1 point**

Trap: "Because R is in front of the speakers." Being in front of the speakers does not make a point loud; P and Q are also in front of them.

(b) **3 points**

For the path lengths to P read from the grid: $8d$ from S_1 and $\sqrt{(8d)^2 + (6d)^2} = 10d$ from S_2, a path difference of $2d$ **1 point**

For using the condition for the first point of destructive interference, $\Delta\ell = \frac{1}{2}\lambda$ (for example, because P is the first quiet point after R, where $\Delta\ell = 0$) **1 point**

For $\lambda = 4d$ **1 point**

Trap: Setting $2d = \lambda$ (the condition for constructive interference), or using the distance from R to P ($3d$) as the path difference.

(c) **1 point**

For selecting "Destructive" and correctly explaining that U is $2d$ from S_1 and $8d$ from S_2, a path difference of $6d = \frac{3}{2}\lambda$ (with the wavelength from (b)), so the waves arrive out of phase (no credit for this part without the correct selection) **1 point**

Credit a selection consistent with an incorrect wavelength from (b) if the path difference $6d$ *is correct and compared correctly with that wavelength. The wave from* S_1 *is stronger at U because U is much closer to* S_1*, so the cancellation is far from complete; U is still a point of destructive interference. Every point on the line beyond* S_1 *has the same path difference,* $6d$*, the separation of the speakers.*

Trap: "Constructive, because U is right next to a speaker." Closeness to one speaker makes that wave strong; it does not decide whether the two waves add or cancel.

(d)(i) **1 point**

For identifying the correct aspect, that the speakers will send out half as much sound energy, with a reason: the speakers are identical and driven by the same generator, so each emits the same power, and one of the two is switched off **1 point**

(d)(ii) **2 points**

For identifying the incorrect aspect, the prediction that the tone at P will get quieter, with the reason that P is a point of destructive interference: the two waves arrive there out of phase and partly cancel **1 point**

For explaining that with S_2 unplugged there is nothing to cancel the wave from S_1, so the amplitude at P increases and the tone gets louder **1 point**

Example response: It's true that one speaker sends out half as much sound energy as two. But P is a quiet spot because the waves from the two speakers arrive out of phase and cancel. Unplug S_2 and nothing cancels the wave from S_1, so the tone at P gets louder.

Trap: Agreeing with the student. Adding a second source can make some places quieter.

(e) **1 point**

For selecting "Louder than" and correctly explaining that doubling the frequency halves the wavelength ($v = f\lambda$ with the speed of sound unchanged), to $2d$; the path difference at P is still $2d$, now one whole wavelength, so the waves arrive in phase and interfere constructively (no credit for this part without the correct selection) **1 point**

Trap: "About as loud, because the speakers and P haven't moved." Whether the waves arrive in phase depends on the path difference measured in wavelengths, and the wavelength changed.

(f) **1 point**

For selecting "Wider" and correctly explaining that a lower frequency means a longer wavelength ($v = f\lambda$, with the speed of sound unchanged), and a wave spreads out (diffracts) more after passing through an opening when its wavelength is larger compared with the width of the opening (no credit for this part without the correct selection) **1 point**

Trap: "The same, because the doorway didn't change." How far a wave spreads after an opening depends on the wavelength compared with the opening, not on the opening alone.

(g) **2 points**

For explaining that the sound's wavelength is comparable to the width of the doorway ($4d$ compared with $8d$, from part (b)), so sound diffracts noticeably through the doorway and spreads into the hallway, reaching places not in a straight line from the speaker **1 point**

For explaining that visible light's wavelength (hundreds of nanometers) is extremely small compared with the doorway, so light passing through the doorway spreads out by a negligible amount and travels in nearly straight lines; the person, off to one side, is not in a straight line through the doorway from the speaker, so no light from the speaker reaches the person's eyes **1 point**

A qualitative comparison (the sound's wavelength is comparable to the width of the doorway) earns the first point; the values from part (b) are not required, and a student with a wrong wavelength in (b) is not penalized here. The wavelength of visible light, about 400–700 *nm, is not given; "much smaller than the door" is enough without a number. For the second point, stating that light travels in nearly straight lines and so does not bend around the edge of the doorway to the person is enough; restating that the person is out of the straight-line path is not required.*

Trap: "Sound goes through walls but light doesn't." The sound reaching the person comes through the open doorway; the question is why it spreads around the edge and light does not.

Chapter 12a: Electric charge, field, and potential

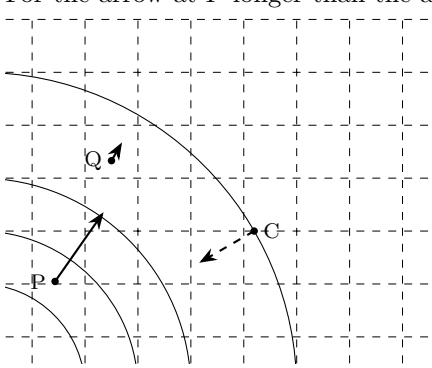
Question 12a.1 (Equipotential map, page 90) **10 points**

(a) **3 points**

For both arrows perpendicular to the equipotential lines **1 point**

For both arrows pointing toward lower potential (away from the lower-left corner) **1 point**

For the arrow at P longer than the arrow at Q **1 point**



Courtesy figure (the dashed arrow at C is part (c)(i)). Near P the potential drops $2V_0$ *over about 1 cm; near Q it drops* V_0 *over about 2 cm, so the field at P is about four times stronger. Only "P longer" is graded.*

(b)(i) **1 point**

For selecting "The same as" and correctly explaining that the object is moved slowly, so the work done by the hand equals the change in the electric potential energy, $q\,\Delta V$, which depends only on the potentials at A and B and not on the path (no credit for this part without the correct selection) **1 point**

Trap: "Greater than: path 2 is longer."

(b)(ii) **2 points**

For relating the hand's work to the electric potential energy: the object is moved slowly, so its kinetic energy does not change and $W_{\text{hand}} = \Delta U_E = q\,(V_B - V_A)$ (equivalently, $W_{\text{hand}} = -W_{\text{field}}$) **1 point**

For $W = q(2V_0 - 4V_0) = -2qV_0$ **1 point**

Trap: Writing $+2qV_0$. The hand holds back a positive charge that the field pushes toward lower potential, so the hand does negative work.

(c)(i) **1 point**

For an arrow at C perpendicular to the equipotential line, pointing toward higher potential (toward the lower-left corner), with the reason that the force on a negative charge is opposite the field **1 point**

(c)(ii) **3 points**

For applying conservation of energy with the electron starting from rest: $\frac{1}{2}mv^2 - 0 = -\Delta U_E$ **1 point**

For $\Delta U_E = q\,\Delta V = (-e)(4V_0 - 2V_0) = -2eV_0$ **1 point**

For $v = \sqrt{\frac{4eV_0}{m}} = 2\sqrt{\frac{eV_0}{m}}$ **1 point**

Trap: A sign slip with $q = -e$ makes the kinetic energy negative. A student who then drops the sign to get a real speed has the right answer for the wrong reason; the second point requires the correct sign of ΔU_E.

A magnitude route earns all three points: the electron moves to higher potential, so its electric potential energy decreases by $e(4V_0 - 2V_0) = 2eV_0$, and that energy becomes kinetic energy, $\frac{1}{2}mv^2 = 2eV_0$.

Question 12a.2 (Charged sphere, three regions, page 91) 10 points

(a)(i) **2 points**

For relating a region's potential difference to the field there, by any one route: it is the area under the E vs. r graph over that region; or it is approximately the average field times the width, $\Delta V \approx E_{\text{avg}}\Delta r$ (equivalently, the work done on a test charge q is about $qE_{\text{avg}}\Delta r$). (Going on to compare regions, e.g., that of equal-width regions the one with the stronger field has the larger potential difference, is acceptable but not required here.) **1 point**

For reasoning that doubling the field strengths throughout a region (the region keeping its width) doubles that region's potential difference, because the width is unchanged while every field strength in it is doubled (the further conclusion that $\Delta V_{D\to A}$ increases by that region's original potential difference is acceptable but not required here) **1 point**

Example response: The potential difference across a region is about the average field strength in the region times the region's width, $\Delta V \approx E_{\text{avg}}\Delta r$. If every field strength in the region is doubled, the average field doubles while the width stays the same, so that region's potential difference doubles.

(a)(ii) **1 point**

For selecting "Region I only" and correctly explaining that region I has the largest potential difference of the three, because the regions are equally wide while the field outside the sphere decreases with distance ($E \propto 1/r^2$), so the field is strongest in region I (or from $V = kQ/r$: $\frac{kQ}{2R}$, $\frac{kQ}{6R}$, $\frac{kQ}{12R}$ for regions I, II, III); doubling it therefore adds the most (no credit for this part without the correct selection) **1 point**

Trap: "Region III, because it is farthest from the sphere," or "All the same, because the regions are the same width." Equal widths matter only if the field were uniform.

(b) **1 point**

For selecting "Consistent" and correctly explaining that the sketch shows the field strongest just outside the surface and decreasing outward, so the potential difference across region I is the largest, as the reasoning in (a)(ii) requires (e.g., a stronger field over an equal width, or the largest area under the curve) (no credit for this part without the correct selection) **1 point**

The zero field inside the sphere does not affect the answer, because all four points are at or outside the surface.

(c) **1 point**

For selecting "Inconsistent" and correctly explaining, by either route: the kinetic energy gained over a short distance is $qE\,\Delta r$, so the slope of the K vs. r graph is proportional to E, and since E is largest just outside the sphere, K should rise most steeply at $r = R$, but the sketch is flat at R and steepest farther out; or the kinetic energy gained across each region is $q\Delta V$ for that region, so by (a)(ii) the gain across region I should be the largest (by $V = kQ/r$, the gains are in the ratio $6 : 2 : 1$), but the sketch gains less across region I than across region II (no credit for this part without the correct selection) **1 point**

In the sketch, K rises by about 1.1, 1.8, and 0.8 units (on the drawing's scale) across regions I, II, and III.

Trap: "Consistent, because K increases with r." The direction is right; the shape near the sphere is wrong.

(d) **3 points**

For applying energy conservation with the potential outside the sphere: $K = q(V_R - V_r) = kqQ\left(\frac{1}{R} - \frac{1}{r}\right)$, where $V_R = \frac{kQ}{R}$ is the potential at the surface (or the equivalent with $U = \frac{kqQ}{r}$) **1 point**

For $K_B = \frac{kqQ}{2R}$ and $K_D = \frac{3kqQ}{4R}$ (with $k = \frac{1}{4\pi\varepsilon_0}$) **1 point**

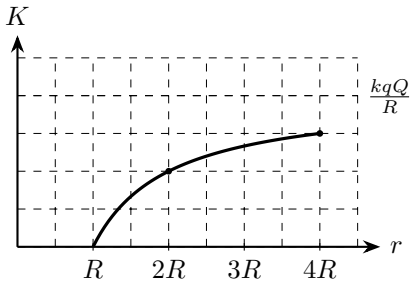
For reasoning that the kinetic energy gained across region I is $K_B - 0 = K_B$, so the fraction is $K_B/K_D = \frac{2}{3}$ (consistent with the student's expressions) **1 point**

Trap: Using $V \propto 1/r^2$, or taking $K = qV$ at the point instead of q times the potential difference from the surface.

(e) **2 points**

For a curve that starts at $K = 0$ at $r = R$, increases throughout, and is consistent with the values in (d) (K at $2R$ is two-thirds of K at $4R$) **1 point**

For a curve that is steepest at $r = R$ and becomes less steep as r increases (concave down), leveling off toward $\frac{kqQ}{R}$; drawing or labeling this limiting value is not required **1 point**



Courtesy figure, drawn as $K = \frac{kqQ}{R}\left(1 - \frac{R}{r}\right)$ with $\frac{kqQ}{R} = 4$ squares: K at $2R$, $3R$, $4R$ is 2, 2.67, 3 squares, so the gains across regions I, II, III are in the ratio $6:2:1$, as in (a)(ii). The curve need not reach $\frac{kqQ}{R}$ on the axes; it should not reach or exceed it by $r = 4R$.

Chapter 12b: Circuits and capacitors

Question 12b.1 (Capacitor with two resistors, page 94) — 12 points

(a) — **1 point**

For selecting "Greater than" and correctly explaining that just after closing the uncharged capacitor has no potential difference across it, so current passes through the capacitor branch and the circuit's resistance is only R_1 (all of the emf is across R_1), while a long time later the capacitor branch carries no current, so the resistance is $R_1 + R_2$ and the current is smaller (no credit for this part without the correct selection) — **1 point**

Example response: Just after closing, current can go through the uncharged capacitor instead of R_2, so the circuit's resistance is only R_1. Later the capacitor passes no current, so the resistance is $R_1 + R_2$ and the current is smaller.

The part says "briefly." A response need not name the loop rule or write $I_1 = \mathcal{E}/R_1$; any correct comparison of the resistance, or of the potential difference across R_1, at both times earns the point. A response that describes only one of the two times does not.

Trap: Treating the capacitor as a break in the circuit at $t = 0$ (the long-time behavior) gives "The same as."

(b)(i) — **2 points**

For explaining that a long time after closing, the capacitor is fully charged and no current flows in its branch, so R_1 and R_2 are in series and carry the same current, which is not zero because there is a complete path through them; therefore there is a nonzero potential difference across R_1 — **1 point**

For explaining that the capacitor is in parallel with R_2, so its potential difference equals that across R_2, and around the loop the emf is shared between R_1 and R_2; since part of the emf is across R_1, the capacitor gets less than $\mathcal{E}$ — **1 point**

Example response: Long after closing, no current goes into the capacitor, but current still flows from the battery through R_1 and then R_2. Because there is current in R_1, there is a potential difference across it. The battery's emf is split between R_1 and R_2, and the capacitor, being in parallel with R_2, has only R_2's share, which is less than $\mathcal{E}$.

Trap: "The capacitor charges until its potential difference equals the battery's emf." That is true for a capacitor in series with a resistor and no other path; here the steady current through R_2 keeps a potential difference across R_1.

(b)(ii) — **1 point**

For selecting "Greater than" and correctly explaining that R_1 and the new resistor share the emf in proportion to their resistances, so the larger resistor takes a larger share (or: the current is smaller, so the potential difference across R_1 is smaller and more of the emf is left across the resistor), and the capacitor, in parallel with that resistor, has a larger potential difference and so holds more charge (no credit for this part without the correct selection) — **1 point**

Example response: With $2R_2$ in place, R_2 takes a larger share of the emf. The capacitor is in parallel with it, so it has that larger potential difference and therefore holds more charge.

An explanation that consists of manipulating $Q = \frac{C\mathcal{E}R_2}{R_1+R_2}$ earns no credit, because the part says not to use equations.

(c) — **3 points**

For the current long after closing: $I = \dfrac{\mathcal{E}}{R_1 + R_2}$, using zero current in the capacitor branch (or, by the voltage-divider route, for treating R_1 and R_2 as in series because the capacitor branch carries no current, so the emf divides in proportion to the resistances) — **1 point**

For the potential difference across the capacitor equal to that across R_2: $\Delta V_C = IR_2 = \dfrac{\mathcal{E}R_2}{R_1 + R_2}$ — **1 point**

For $Q = C\,\Delta V_C = \dfrac{C\mathcal{E}R_2}{R_1 + R_2}$ — **1 point**

(d) — **2 points**

For identifying $\Delta V_C = \frac{\mathcal{E}R_2}{R_1+R_2}$ (or $\Delta V_{R_2} = IR_2$) as the step that shows R_2's share of the emf — **1 point**

For explaining that the fraction $\frac{R_2}{R_1+R_2}$ increases when R_2 increases, so the capacitor's potential difference and charge increase, as argued in (b)(ii) — **1 point**

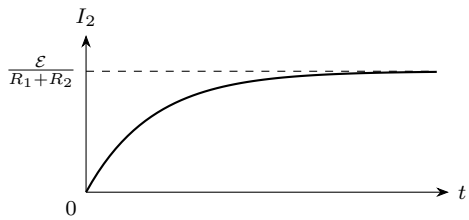
Score (d) against the student's own (b)(ii) and (c). If (b)(ii) argued from the current instead (larger R_2, smaller current, but $\Delta V_{R_2} = IR_2$ still grows), then pointing to $I = \frac{\mathcal{E}}{R_1+R_2}$ together with $\Delta V = IR_2$ earns both points. If (c) used Kirchhoff's loop rule, pointing to $\mathcal{E} - IR_1 - \Delta V_C = 0$ and arguing that IR_1 shrinks earns both points.

(e) **3 points**

For a curve that starts at $I_2 = 0$ at $t = 0$ **1 point**

For a curve that increases with decreasing slope and levels off, without overshooting **1 point**

For labeling the value the curve approaches as $\dfrac{\mathcal{E}}{R_1 + R_2}$ **1 point**



Courtesy plot. The points attach to the features described in words; the exact curvature is not graded. I_2 starts at zero because the uncharged capacitor, in parallel with R_2, puts no potential difference across R_2; no explanation is required.

Trap: Sketching the familiar decaying capacitor-current curve, which is the current in R_1 (or in the capacitor), not in R_2.

Question 12b.2 (Conductive dough, page 96) 12 points

(a) **4 points**

For rolling the same lump (none added or removed) into uniform cylinders of several different lengths **1 point**

For measuring each length with the meterstick **1 point**

For measuring the resistance between the foil disks with the multimeter on its resistance setting (or using it to measure current and potential difference with a source, and computing $R = \frac{\Delta V}{I}$) **1 point**

For a way to reduce uncertainty: several lengths, repeated measurements at each length, or checking uniform thickness with calipers **1 point**

(b) **2 points**

For using $R = \frac{\rho \ell}{A}$ with $A = \frac{V_d}{\ell}$ because the volume is fixed **1 point**

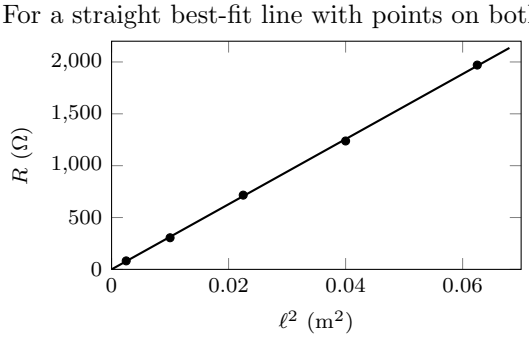
For $R = \dfrac{\rho\, \ell^2}{V_d}$ **1 point**

(c) **3 points**

For choosing R versus ℓ^2 (or another pair consistent with the student's answer to (b) that should be linear) **1 point**

For labeled axes with units, scales that use more than half of the grid, and all points plotted correctly **1 point**

For a straight best-fit line with points on both sides **1 point**



Courtesy plot. ℓ^2 values: 0.0025, 0.0100, 0.0225, 0.0400, 0.0625 m^2. The best-fit slope is about $3.1\times10^4\ \Omega/m^2$.

(d) **1 point**

For selecting "No" and correctly explaining from the data that doubling the length roughly quadruples the resistance rather than doubling it (for example, 0.100 m to 0.200 m takes R from 305 Ω to 1238 Ω), or that R versus ℓ curves while R versus ℓ^2 is straight (no credit for this part without the correct selection) **1 point**

Trap: The claim holds for a wire whose thickness is fixed. Here, rolling the same lump longer also makes it thinner; reading "same lump" carefully is the point of the question.

(e) **2 points**

For relating the slope to the resistivity: slope $= \frac{\rho}{V_d}$, so $\rho = (\text{slope})\,V_d$ **1 point**

For a value consistent with the student's line, about 0.25 Ω · m (accept 0.22–0.28 Ω · m) **1 point**

Chapter 13: Magnetism

Question 13.1 (Proton-therapy bending magnet, page 98) 10 points

(a) **2 points**

For a path that curves toward the top of the page (the force on a positive charge moving right in a field into the page is up the page, $\vec{F} = q\vec{v}\times\vec{B}$) **1 point**

For a circular arc that starts tangent to the entering direction and is a half circle, ending at the boundary with the proton moving back to the left **1 point**

(b) **3 points**

For energy conservation in the accelerating stage: $eV = \frac{1}{2}mv^2$, so $v = \sqrt{2eV/m}$ **1 point**

For Newton's second law with the magnetic force as the centripetal force: $evB = \dfrac{mv^2}{r}$ **1 point**

For $r = \dfrac{1}{B}\sqrt{\dfrac{2mV}{e}}$ (or $\dfrac{\sqrt{2meV}}{eB}$) **1 point**

(c) **1 point**

For selecting "Greater than" and correctly justifying it: from (b), $r \propto \sqrt{m/q}$ at the same V and B, and m/q is twice as large for the alpha particle ($4m/2e$), so its radius is larger (by a factor of $\sqrt{2}$; the factor is not required); or equivalent reasoning about the alpha particle's speed ($\frac{1}{\sqrt{2}}$ times the proton's) and the force on it, carried through to the radius (no credit for this part without the correct selection) **1 point**

Trap: "Less than: twice the charge means twice the force, so a tighter turn." The alpha particle also gains twice the energy and has four times the mass.

(d) **1 point**

For selecting "Stay the same" and correctly explaining that the radius is proportional to the speed ($evB = mv^2/r$, so $r = mv/(eB)$), so a faster proton travels a proportionally longer half circle, and the time $\pi r/v = \pi m/(eB)$ does not depend on the speed; or that the angular speed $\omega = v/r = eB/m$ does not depend on v and the proton always turns through half a revolution (no credit for this part without the correct selection) **1 point**

Trap: "Decrease, because the proton is faster." It is faster, but its path is longer in proportion.

(e) **3 points**

For (i): $eV = \frac{1}{2}mv^2$, so $v = \sqrt{2eV/m} = 2.0\times10^7$ m/s **1 point**

For (ii): the distance is the diameter, $2r = \dfrac{2mv}{eB}$ (or twice the result of (b)) $= 0.27$ m **1 point**

For (iii): $t = \dfrac{\pi r}{v}$ (or $\dfrac{\pi m}{eB}$) $= 2.2\times10^{-8}$ s **1 point**

Values consistent with an incorrect speed from (i) earn the points for (ii) and (iii). The proton's speed is about 7% of the speed of light, so the nonrelativistic treatment is adequate.

Trap: Giving the radius (0.14 m) for the distance in (ii). The proton leaves the field one diameter from where it entered.

Question 13.2 (Two extension cords, page 100) **12 points**

(a) **1 point**

For "Into the page" (right-hand rule: with the current in cable 1 to the right, the field below the cable points into the page) **1 point**

(b) **1 point**

For selecting "Repel" and correctly justifying it, for example: the force on cable 2 (current to the left) in cable 1's field, which is into the page, is $\vec{F} = I\vec{\ell} \times \vec{B}$, toward the bottom of the page, away from cable 1; or: currents in opposite directions repel (no credit for this part without the correct selection) **1 point**

(c)(i) **2 points**

For identifying a correct aspect: the force on a current-carrying wire is proportional to its own current (and to the field it is in), or cable 2's field at cable 1 is three times as strong as cable 1's field at cable 2. Merely restating the given fact that cable 2 carries three times the current does not earn this point. **1 point**

For a correct reason tied to the aspect identified: for the force claim, $F = I\ell B$ shows that the force on a wire in a given field is proportional to that wire's current; for the field claim, $B = \dfrac{\mu_0 I}{2\pi r}$ at the same distance with three times the current. Writing an equation without connecting it to the claim does not earn this point. **1 point**

(c)(ii) **2 points**

For identifying the incorrect aspect: the conclusion that the two forces have different magnitudes **1 point**

For explaining that the forces form a Newton's-third-law pair, so they are equal in magnitude; or showing it: the force on cable 1 is $I\ell \cdot \frac{\mu_0(3I)}{2\pi s}$ and the force on cable 2 is $3I\ell \cdot \frac{\mu_0 I}{2\pi s}$, which are equal. The student counted the factor of 3 in one place and missed it in the other. **1 point**

Trap: Agreeing with the student because "more current means more force." Each cable's force depends on both currents.

(d) **3 points**

For the field of cable 2 at cable 1, $B_2 = \dfrac{\mu_0(3I)}{2\pi s}$ **1 point**

For the force on a length of cable 1, $F = I\ell B_2$ (cable 1's own current, not $3I$) **1 point**

For $F = \dfrac{3\mu_0 I^2 \ell}{2\pi s}$ **1 point**

(e) **1 point**

For a factor of $\frac{1}{2}$, with the reasoning that each cable's field at the other is inversely proportional to the distance ($B = \frac{\mu_0 I}{2\pi r}$) while the currents are unchanged, so $F = I\ell B$ is halved (or from the expression in (d) with $2s$ in place of s) **1 point**

Trap: A factor of $\frac{1}{4}$, from treating the wires like point charges with an inverse-square law.

(f) **2 points**

For reasoning that the two fields at P point in the same direction, into the page (cable 1's current is to the right and P is below it; cable 2's current is to the left and P is above it), so their magnitudes add **1 point**

For $B_P = \dfrac{\mu_0 I}{2\pi(s/2)} + \dfrac{\mu_0(3I)}{2\pi(s/2)} = \dfrac{4\mu_0 I}{\pi s}$, into the page **1 point**

Trap: Subtracting the two fields ($\frac{2\mu_0 I}{\pi s}$) because the currents are in opposite directions. Opposite currents on opposite sides of P give fields in the same direction there.

Chapter 14: Electromagnetic induction

Question 14.1 (Copper disk and magnet, page 102) — 10 points

(a) — **2 points**

For the magnitude: the electron moves in a circle with speed $v = \omega r$, perpendicular to the field, so $F = evB = e\omega rB$ — **1 point**

For "toward the axis," with a correct right-hand-rule argument: the electron's velocity is tangent to its circle, $\vec{v} \times \vec{B}$ points away from the axis, and the electron's negative charge reverses the force — **1 point**

Trap: Forgetting that the electron is negative and answering "away from the axis."

(b) — **3 points**

For the balance condition stated in words or symbols: the electric force on each free electron is equal in magnitude and opposite in direction to the magnetic force from (a), $eE = e\omega rB$ — **1 point**

For $E = \omega rB$ (consistent with the magnitude found in (a)) — **1 point**

For "toward the axis," with the reason: the electric force on each electron must point away from the axis to balance the magnetic force, and the electron is negative, so the field points toward the axis (or: electrons collect near the center, leaving the rim positive, so the field points from rim to center) — **1 point**

Trap: Including the centripetal force on the electron. It is real but negligible here, and the question states the balance to use.

(c) — **1 point**

For selecting "Equal to twice" and correctly explaining that doubling ω doubles each electron's speed ωr and so doubles the magnetic force, and hence the balancing electric field, at every distance from the axis; the field along the whole path from A to B is doubled, so the potential difference is doubled (no credit for this part without the correct selection) — **1 point**

Responses may instead use $\Delta V = \frac{1}{2}\omega B(r_B^2 - r_A^2)$ (area under E vs. r, or calculus) to show the proportionality; that relationship is not required.

Trap: "Greater than twice, because the field grows with r." The field's growth with r is the same at both angular speeds; only its overall size doubles.

(d) — **1 point**

For selecting "Dot B" and correctly explaining that the magnetic force pushes the free electrons toward the axis (from (a)), so negative charge collects near the center and the outer part of the disk is left positive; the potential is therefore higher farther from the axis (equivalently, the electric force on each electron must point away from the axis to balance the magnetic force, so the electric field points toward the axis, and potential decreases in the direction of the field) (no credit for this part without the correct selection) — **1 point**

(e) — **1 point**

For selecting "Less than" and correctly explaining that A to C and C to B span equal distances along the radius, but the electric field, $E = \omega Br$, is larger at every point between C and B than between A and C, so moving a charge from C to B takes more work per unit charge than moving it the same distance from A to C, and the potential difference is larger from C to B (any correct route, e.g., average field times distance, area under the E vs. r graph, or $r_C^2 - r_A^2 < r_B^2 - r_C^2$; none of these relationships is required) (no credit for this part without the correct selection) — **1 point**

Trap: "Equal to, because C is halfway between." Equal distances give equal potential differences only in a uniform field.

Table: Still/Still, Zero; Still/Spinning, Nonzero; Spinning/Still, Zero; Spinning/Spinning, Nonzero. No points are awarded for the table by itself; each explanation below is scored only if the student's choice in its row is correct. Responses that say "emf" for the potential difference within the disk are not penalized.

(f)(i) — **1 point**

For choosing "Zero" for the spinning magnet and still disk and correctly explaining that the magnet's field is symmetric about the axis, so spinning the magnet does not change the field at any point of the disk, and the disk's charges are at rest in this unchanging field, so the field exerts no force on them ($F = qvB\sin\theta$ with $v = 0$); stating that there is no changing flux or induced electric field is acceptable but not required (no credit for this part without the correct choice in this row) — **1 point**

Trap: "Spinning magnet, still disk: nonzero, because a moving magnet induces an emf." A moving magnet induces an emf only if it changes the field somewhere; spinning a symmetric magnet about its own axis does not.

(f)(ii) **1 point**

For choosing "Nonzero" for both spinning and correctly explaining that the disk's free charges are carried through the field and pushed along the radius by the magnetic force (perpendicular to their velocity and to the field), which separates charge until a sustained potential difference exists between A and B, which are at different distances from the axis; the magnet's spin adds nothing, because its symmetric field at each point of the disk is unchanged, so the result is the same as for a spinning disk and a still magnet, as in parts (a)–(e) (no credit for this part without the correct choice in this row) **1 point**

Trap: "Both spinning: zero, because there is no relative motion." There is no relative motion between the disk and the magnet, but the disk's charges still move through the field, which is the same at every point as when the magnet is still.

Question 14.2 (Three loops into a field, page 105) 12 points

(a) **1 point**

For selecting the ranking $\mathcal{E}_Z > \mathcal{E}_X > \mathcal{E}_Y$ and correctly justifying it: the emf equals the rate of change of flux, $B\times$(rate at which area enters the field) $= Bvw$, where w is the length of the leading edge that crosses the boundary (the loop's height), and the heights are $2a$, a, $a/2$ (or: the emf is that of the moving leading edge, $\mathcal{E} = Bvw$) (no credit for this part without the correct ranking) **1 point**

Trap: Ranking by area (all three loops have area a^2, so that gives a three-way tie) or by the length along the motion. Only the edge crossing the boundary matters.

(b) **3 points**

For the emfs $\mathcal{E} = Bvw$ with the leading-edge lengths $2a$, a, $a/2$ for Z, X, Y, together with $I = \mathcal{E}/R$ **1 point**

For the resistances $R = \lambda\times$perimeter: $5a\lambda$, $4a\lambda$, $5a\lambda$ for Z, X, Y **1 point**

For $I_X = \dfrac{Bv}{4\lambda}$, $I_Y = \dfrac{Bv}{10\lambda}$, $I_Z = \dfrac{2Bv}{5\lambda}$ **1 point**

Trap: Using the same resistance for all three loops. Y and Z have the same perimeter, $5a$, but X has a smaller one, $4a$.

(c) **3 points**

For reasoning that the magnetic forces on the parts of the top and bottom edges inside the field are equal and opposite (equal lengths, opposite currents), so only the leading edge contributes a net magnetic force, $F = I_X aB$, opposite the motion **1 point**

For reasoning that at constant speed the net force is zero, so the applied force equals this magnetic force, with $I_X = \dfrac{Bv}{4\lambda}$ from (b) **1 point**

For $F = \dfrac{B^2av}{4\lambda}$ **1 point**

An energy method (applied force times v equals the power $\mathcal{E}^2/R$) earns full credit.

(d)(i) **1 point**

For selecting "Counterclockwise" and correctly justifying it: the flux into the page through the loop is increasing, so the induced current makes a field out of the page inside the loop, which requires a counterclockwise current (or: the force on positive charges in the moving right edge, $q\vec{v}\times\vec{B}$, points up the page) (no credit for this part without the correct selection) **1 point**

(d)(ii) **1 point**

For zero current, because the flux through the loop no longer changes (or: the emfs in the left and right edges cancel) **1 point**

(e) **3 points**

For the time during which loop Z carries current: it must move its own length along the motion, $a/2$, so $t = \dfrac{a}{2v}$ **1 point**

For the rate of energy dissipation, $P = I_Z^2 R_Z = \dfrac{\mathcal{E}_Z^2}{R_Z} = \dfrac{(2aBv)^2}{5a\lambda}$ (or the work done by the applied force, $F = I_Z(2a)B$, over the distance $a/2$) **1 point**

For $E = Pt = \dfrac{2a^2B^2v}{5\lambda}$ **1 point**

Trap: Using the loop's height ($2a$) as the distance moved while entering. The emf depends on the leading edge; the time depends on the length along the motion.

Question 14.3 (Rod falling on vertical rails, page 107) 10 points

(a) **1 point**

For selecting "From Y to X" and correctly justifying it: the circuit formed by the resistor, the rails, and the rod encloses a growing area as the rod falls, so the magnetic flux into the page through it increases; by Lenz's law the induced current makes a field out of the page inside the circuit, which requires a counterclockwise current, leftward through the resistor (or: the magnetic force $q\vec{v} \times \vec{B}$ on positive charges in the downward-moving rod points to the right, so current flows rightward in the rod, up the right rail, and leftward through the resistor) (no credit for this part without the correct selection) **1 point**

Trap: "From X to Y," from taking the induced field to point into the page (in the same direction as the external field) or from finding that the free electrons in the rod are pushed to the left and taking that as the direction of the current. The flux into the page is increasing, so the induced field inside the circuit points out of the page.

(b) **3 points**

For the induced emf, $\mathcal{E} = B\ell v$ (from the rate of change of flux, $B\ell\,\Delta y/\Delta t$, or as motional emf), and the current $I = \dfrac{B\ell v}{R}$ **1 point**

For the magnetic force on the rod, $F = I\ell B = \dfrac{B^2\ell^2 v}{R}$, directed upward, set equal to mg at constant speed (net force zero) **1 point**

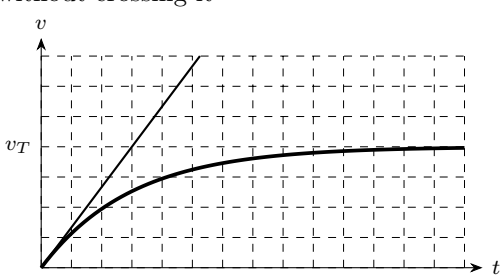
For $v_T = \dfrac{mgR}{B^2\ell^2}$ **1 point**

An energy route also earns full credit: at constant speed the rate of loss of gravitational potential energy, mgv_T, equals the power dissipated in the resistor, $\mathcal{E}^2/R = B^2\ell^2 v_T^2/R$. The second point requires the balance (or the energy equality) to be stated, not just used.

(c) **2 points**

For a curve that starts at $v = 0$ and is initially tangent to the free-fall line (at release the speed, and therefore the current and the magnetic force, is zero, so the acceleration is g) **1 point**

For a curve that then bends below the free-fall line, concave down, and levels off approaching v_T without crossing it **1 point**



Courtesy plot: $v = v_T\left(1 - e^{-gt/v_T}\right)$; students need not know the function. With the data in (d), the free-fall line reaches $v_T = 2.45$ m/s at 0.25 s, and the rod reaches 98% of v_T at about 1.0 s. A curve that overshoots v_T and settles back does not earn the second point; the magnetic force can exceed mg only if $v > v_T$, which the rod never reaches from rest.

Trap: Starting the curve with a slope smaller than the free-fall slope. At the instant of release there is no current, so the rod's initial acceleration is exactly g.

(d) **2 points**

For the power dissipated in the resistor: at constant speed the magnetic force balances the weight, $I\ell B = mg$, so $I = \dfrac{mg}{B\ell} = 0.49$ A and $P = I^2R = 0.48$ W (or $v_T = 2.45$ m/s, $\mathcal{E} = B\ell v_T = 0.98$ V, $P = \mathcal{E}^2/R = 0.48$ W) **1 point**

For the rate of decrease of gravitational potential energy, $mgv_T = (0.020\text{ kg})(9.8\text{ m/s}^2)(2.45\text{ m/s}) = 0.48$ W, equal to the power in the resistor **1 point**

With $g = 10$ m/s^2 both are 0.50 W; accept. The first point does not need (b) (force balance gives the current directly). The second point requires mgv_T computed correctly from the student's own v_T (from (b), or from the student's own work here), with the two rates compared honestly: if the student's numbers differ, saying they agree does not earn the point. The kinetic energy is constant, so all the gravitational potential energy lost is converted to internal energy in the resistor; students need not say this.

(e) **1 point**

For selecting "Greater than" and correctly justifying it: at a given speed the emf is the same, so with twice the resistance the current, and therefore the upward magnetic force $I\ell B$, is half as large; the force reaches mg only at twice the speed (or from (b), $v_T \propto R$) (no credit for this part without the correct selection) **1 point**

Trap: "Less than, because a larger resistance opposes the motion more." The resistance limits the current, and it is the current that produces the retarding force.

(f) **1 point**

For selecting "Equal to g" and correctly justifying it: an emf $B\ell v$ is still induced across the rod, but with the gap there is no complete circuit, so there is no current (the brief, tiny shift of charge to the ends of the rod is negligible), and with no current there is no magnetic force on the rod; only gravity acts (no credit for this part without the correct selection) **1 point**

Trap: "Less than g, because an emf is still induced." An emf produces a retarding force only when it drives a current.

Chapter 15: Physical optics

Question 15.1 (Laser and a hair, page 110) — 12 points

(a) **1 point**

For selecting "Decrease" and correctly justifying it: the first dark fringe is at $\sin\theta = \lambda/a$, so a larger a gives a smaller angle, and at the same distance L the dark fringes on either side are closer together, so the central fringe is narrower (no credit for this part without the correct selection) **1 point**

Trap: "Increase: a bigger obstacle makes a bigger pattern." Diffraction spreads light more when the obstacle or slit is narrower.

(b) **3 points**

For the single-slit dark-fringe condition $a\sin\theta = \lambda$ for the first dark fringe **1 point**

For the small-angle geometry: distance from center $\approx L\tan\theta \approx L\sin\theta$, so each first dark fringe is $\frac{\lambda L}{a}$ from the center **1 point**

For $W = \dfrac{2\lambda L}{a}$ **1 point**

Trap: Forgetting the factor of 2: W spans from one first-order dark fringe to the other.

(c) **3 points**

For correctly scaled and labeled axes (with units) that use more than half the grid **1 point**

For correctly plotted points **1 point**

For a reasonable best-fit line through or near the origin **1 point**

(d) **2 points**

For using the slope, about 16 mm/m = 0.016 (accept 0.015–0.017), not two data points far from the line, and relating it to a: slope $= \dfrac{2\lambda}{a}$ **1 point**

For $a \approx 8 \times 10^{-5}$ m (80 μm; accept 75–87 μm) **1 point**

Score (d) for consistency with the expression in (b). Using slope $= \lambda/a$ gives about 40 μm and earns the second point only if (b) lacked the factor of 2.

(e) **2 points**

For a valid feature: measuring between the two dark fringes doubles the measured distance and avoids locating the center of a broad bright fringe; or using several values of L and the slope of a best-fit line averages out random errors in individual measurements **1 point**

For explaining why that reduces uncertainty (a fixed reading error is a smaller fraction of a larger distance; the best-fit line is less affected by any one bad measurement) **1 point**

(f) **1 point**

For using $W = \dfrac{2\lambda L}{a}$ with the value of a from (d) (or scaling the measured or best-fit W at $L = 2.0$ m by $\frac{532}{650}$, since $W \propto \lambda$ at fixed L and a) to get $W \approx 27$ mm (accept 24–29 mm, or a value consistent with the student's a) **1 point**

Question 15.2 (Soap film and lens coating, page 112) 10 points

(a) **2 points**

For identifying the two reflected waves that interfere, one from the front (air–soap) surface and one from the back (soap–air) surface, and reasoning that only the front reflection is inverted (shifted by half a wavelength), because reflection from a medium of higher index inverts the wave and reflection from a medium of lower index does not **1 point**

For reasoning that when the film is much thinner than the wavelength, the extra path traveled by the back reflection is negligible, so the two reflected waves are half a wavelength out of step for every color and interfere destructively **1 point**

Example response: Light reflects from both the front and the back of the film. At the front, light reflects off soap, which has a higher index than air, so that wave is flipped; at the back, it reflects off air, which has a lower index, so that wave is not flipped. Where the film is much thinner than a wavelength, the back wave travels almost no extra distance, so the only difference between the waves is the flip. They cancel for every color, and the film looks black.

(b) **1 point**

For selecting "No" and correctly explaining that both reflections are now from a medium of higher index (air to coating, $1.00 \to 1.38$, and coating to glass, $1.38 \to 1.52$), so both reflected waves are inverted, and with negligible extra path they are in step and interfere constructively, so the coating reflects (much like uncoated glass) rather than looking black (no credit for this part without the correct selection) **1 point**

Saying only that a coating much thinner than the wavelength behaves like bare glass, which reflects, without the comparison of indices at both surfaces, is not a correct explanation.

Trap: "Yes, any very thin film looks black." That rule depends on exactly one of the two reflections being inverted, which is true for a soap film in air but not for this coating.

(c) **3 points**

For reasoning that since both reflections are inverted, destructive interference requires the extra round-trip path in the coating to be half a wavelength in the coating **1 point**

For using the wavelength in the coating, $\lambda/1.38$: $2t = \dfrac{1}{2} \cdot \dfrac{\lambda}{1.38}$ (equivalently, $2n_c t = \dfrac{\lambda}{2}$ with $n_c = 1.38$, which earns this point) **1 point**

For $t = \dfrac{\lambda}{4(1.38)} \approx 0.18\lambda$ **1 point**

Trap: Using the soap-film condition (one inversion) gives $t = \frac{\lambda}{2(1.38)}$, which makes the coating reflect more, not less.

(d) **2 points**

For reasoning that the thickness makes the extra round-trip path, $2t$, exactly half a wavelength (in the coating) only for 550 nm, so only green light is cancelled **1 point**

For reasoning that for red and violet light, $2t$ is a different fraction of the wavelength in the coating, so the two reflected waves are not exactly half a wavelength out of step and cancel only partly; the reflected light is mostly the ends of the spectrum, red and violet, which together look purple **1 point**

Trap: "The coating absorbs green light." The green light is not reflected because of interference; it is transmitted into the lens.

(e) **2 points**

For reasoning that only one reflection is inverted (as in (a)), so constructive interference requires the extra round-trip path to be half a wavelength in the film: $2t = \frac{1}{2} \cdot \frac{\lambda}{1.33}$ **1 point**

For $t = \frac{600 \text{ nm}}{4(1.33)} \approx 113$ nm (1.1×10^{-7} m) **1 point**

Trap: Using the condition $2t = \lambda/n$ (no inversion), which gives a dark band, or forgetting to use the wavelength in the film (giving 150 nm).

Chapter 16: Geometric optics

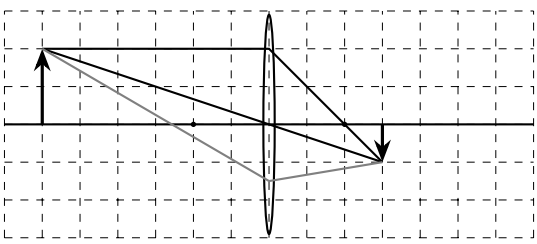
Question 16.1 (Half-covered magnifying glass, page 114) 10 points

(a) **3 points**

For a ray from the top of the window parallel to the axis that refracts through the far focal point **1 point**

For a second correct ray (through the center of the lens undeviated, or through the near focal point and then parallel to the axis) **1 point**

For an inverted image drawn where the rays cross, about $1.5f$ beyond the lens and half as tall **1 point**



Courtesy figure (grid units: f = 2 squares); the gray ray is the one asked for in (c). Points attach to the features described in words.

(b) **3 points**

For using the thin-lens equation, $\frac{1}{3f} + \frac{1}{d_i} = \frac{1}{f}$ **1 point**

For $d_i = \frac{3f}{2}$ **1 point**

For the height ratio $\frac{d_i}{d_o} = \frac{1}{2}$ (magnification $-\frac{1}{2}$, inverted) **1 point**

(c) **1 point**

For a ray from the top of the window to the lens below the axis, refracted so that it passes through the top of the (inverted) image drawn in (a), or through the image point consistent with the student's image **1 point**

With the grid in (a), the ray goes from the top of the window, $(-6, 2)$*, to the lens at about* $(0, -1.5)$ *and then through the tip of the image at* $(3, -1)$*.*

(d) **1 point**

For selecting "The whole image remains, but it is dimmer" and correctly explaining that light from every point of the window strikes every part of the lens, and the rays from each point that pass through the uncovered half still converge to the same image point (as in (c), rays through either half reach the same point), so every part of the image still forms in the same place; the cardboard blocks some of the rays from each point, so less light reaches each image point and the image is dimmer (no credit for this part without the correct selection) **1 point**

Trap: "The top (or bottom) half disappears." Each half of the lens forms a complete image; the ray diagram uses only two of the many rays from each point.

(e) **2 points**

For reasoning that an image the same size requires a magnification of magnitude 1, so $d_i = d_o$, and using the thin-lens equation, $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ **1 point**

For $d_o = d_i = 2f$: the window is $2f$ from the lens, and the wall is $2f$ from the lens **1 point**

Trap: Placing the window at f to get a "same-size" image. An object at the focal point forms no image on a wall.

Question 16.2 (Coin in a mug, page 116) 12 points

(a) **2 points**

For a ray from the coin to the surface at the normal line, then from the surface to the eye, with the direction of travel shown from the coin toward the eye **1 point**

For the ray in air making a larger angle with the normal than the ray in the tea (bending away from the normal) **1 point**

(b)(i) **2 points**

For identifying a correct aspect: the light bends (refracts) at the surface, or it changes speed there **1 point**

For a correct reason matched to the aspect identified: for the speed change, light travels at different speeds in tea and air ($v = c/n$); or, for the bending, Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$, with $n_1 \neq n_2$ requires a change of direction for a slanted ray **1 point**

(b)(ii) **2 points**

For identifying an incorrect aspect: light leaving the tea bends away from the normal (not toward), or the coin looks shallower (not deeper) **1 point**

For explaining: going from larger n to smaller n, Snell's law gives a larger angle in air; tracing the bent rays back into the tea, they appear to come from a point above the coin, so the coin looks closer to the surface **1 point**

(c) **3 points**

For Snell's law at the surface: $n \sin\theta_t = \sin\theta_a$ **1 point**

For the geometry: the ray meets the surface a horizontal distance $x = d\tan\theta_t = d_{\text{app}}\tan\theta_a$ from the point above the coin, and for small angles $\tan\theta \approx \sin\theta$ **1 point**

For $d_{\text{app}} = \dfrac{d}{n}$ **1 point**

(d) **1 point**

For selecting "Less than" and correctly justifying it: from (c), $d_{\text{app}} = d/n$, so a greater n at the same depth gives a smaller apparent depth; or: going from a larger index into air, the rays bend more, so their backward extensions meet higher, closer to the surface (no credit for this part without the correct selection) **1 point**

(e) **2 points**

For identifying the edge of the region as the point where the ray from the coin meets the surface at the critical angle, $n \sin\theta_c = 1$ (the refracted ray would travel along the surface), so $\sin\theta_c = \dfrac{1}{n}$ **1 point**

For the geometry, radius $= d\tan\theta_c$, giving $\dfrac{d}{\sqrt{n^2-1}}$ (or the equivalent $d\tan\left[\sin^{-1}(1/n)\right]$) **1 point**

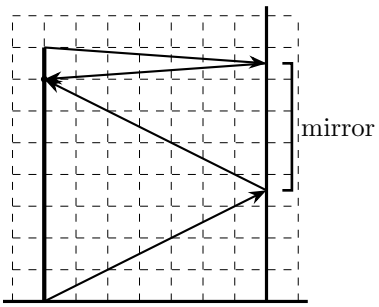
Trap: Using the small-angle result from (c). The critical angle is not small: for tea ($n \approx 1.33$) it is about $49°$, and the radius is about $1.1d$.

Question 16.3 (Bathroom mirrors, page 118) 12 points

(a) **2 points**

For a ray from the feet to the wall and then to the eye that reflects with equal angles to the (horizontal) normal, meeting the wall halfway between the floor and eye level (at a height of 3.5 squares) **1 point**

For a ray from the top of the head to the wall and then to the eye, reflecting with equal angles halfway between eye level and the top of the head (at 7.5 squares), and a bracket along the wall between the two reflection points **1 point**



Courtesy figure. Arrowheads are not required. A ray drawn toward the image of the feet or head behind the wall (the image is as far behind the wall as the person is in front) is an equally good way to find the reflection points. Accept reflection points within about half a square of 3.5 and 7.5 squares.

(b) **2 points**

For the bottom edge: by the law of reflection with a horizontal normal, the ray from the feet reaches the wall halfway between the floor and the eye, at height $\frac{h}{2}$ (equal angles give two congruent right triangles, each with horizontal side D; or the line from the eye to the image of the feet, a distance $2D$ away, crosses the wall halfway) **1 point**

For the top edge at $h + \frac{H-h}{2} = \frac{H+h}{2}$ by the same reasoning, so the smallest height is $\frac{H}{2}$ (independent of D) **1 point**

A bare "$H/2$" or "$h/2$" with no reasoning earns no credit. Reasoning that applies the halfway result once to the whole body (the mirror spans half the distance from the feet to the eye plus half the distance from the eye to the top of the head) earns the second point. An answer that depends on D earns neither point.

Trap: Thinking the mirror must be as tall as the person. Each ray reflects halfway (in height) between the body point and the eye, so the mirror spans half of every height interval.

(c) **1 point**

For selecting "The same as" and correctly justifying it: the reflection points stay halfway between the floor and the eye and halfway between the eye and the top of the head for any distance (the triangles scale but the halfway heights do not change), so the minimum height is still $H/2$, as in (b) (no credit for this part without the correct selection) **1 point**

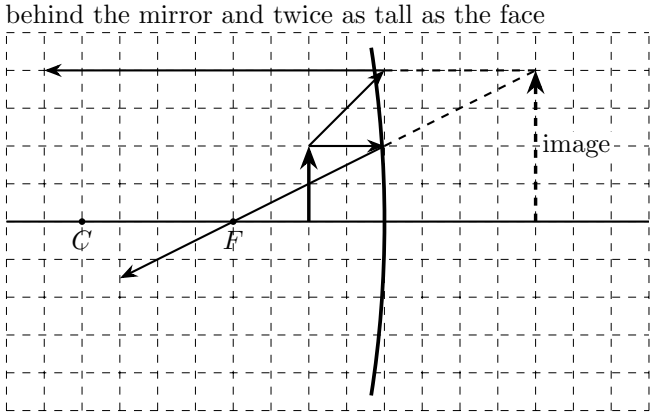
Trap: "Less than: when you back up, you see more of yourself." Backing away makes everything in the mirror look smaller, but the part of the wall that reflects your feet into your eye does not move.

(d) **3 points**

For one correct ray from the tip of the face, reflected correctly: parallel to the axis, then reflected through F; or along the line from F through the tip, then reflected parallel to the axis; or to the center of the mirror, then reflected at an equal angle below the axis; or along the line from C, reflected back on itself **1 point**

For a second correct ray, with both reflected rays diverging in front of the mirror and extended (for example, dashed) behind the mirror until they meet **1 point**

For an upright image drawn behind the mirror where the extensions meet, about f (4 squares) behind the mirror and twice as tall as the face **1 point**



Courtesy figure (grid units: $f = 4$ squares). The ray from the tip of the face through the mirror's center reflects toward $(-4, -4)$; its extension also passes through the image tip $(4, 4)$. The image may be drawn with a solid or dashed line.

Trap: Drawing the image where the reflected rays would cross in front of the mirror. For an object inside F, the reflected rays diverge and never cross in front.

(e) **2 points**

For using the mirror equation, $\frac{1}{f/2} + \frac{1}{d_i} = \frac{1}{f}$, giving $d_i = -f$: the image is a distance f from the mirror, behind it (virtual) **1 point**

For the height ratio $\left|\frac{d_i}{d_o}\right| = \frac{f}{f/2} = 2$ (magnification +2, upright) **1 point**

A geometric derivation from the ray diagram (similar triangles) earns full credit. A sign error that gives $d_i = +f$ earns the first point only if the student still states that the image is behind the mirror with a reason (for example, from the ray diagram); the second point may be earned consistent with the student's $|d_i|$.

(f) **1 point**

For selecting "Greater than" and correctly justifying it: with $d_o = 3f/4$, the mirror equation gives $d_i = -3f$ and magnification $\frac{3f}{3f/4} = 4$ (or in general $M = \frac{f}{f - d_o}$, which grows as d_o approaches f); or, on a ray diagram, the ray parallel to the axis still reflects through F, and the ray from F through the tip of the face now strikes the mirror farther from the axis, so the image tip is higher (no credit for this part without the correct selection) **1 point**

This asks about the image's height, not how large the face looks to her eye. The image also moves much farther behind the mirror, so its apparent (angular) size from her eye actually decreases; a student who argues correctly about angular size instead of image height does not earn the point, since the question asks about height.

Trap: "Less than, because moving away from a mirror makes the image smaller," carried over from plane mirrors. Inside the focal point of a concave mirror, the image height grows as the object approaches F.

(g) **1 point**

For selecting "Less than" and correctly justifying it: a convex mirror has a negative focal length, $\frac{1}{f/2} + \frac{1}{d_i} = -\frac{1}{f}$ gives $d_i = -\frac{f}{3}$ and magnification $\frac{f/3}{f/2} = \frac{2}{3}$; or, by rays, the reflected rays diverge as if from points behind the mirror closer to it than the face is in front, so the upright image is smaller (no credit for this part without the correct selection) **1 point**

Trap: Using $+f$ for the convex mirror, which reproduces the concave result of a magnified image.

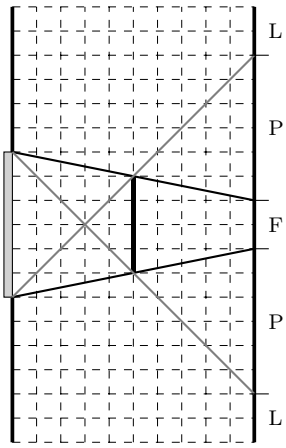
Question 16.4 (Window, board, and shadow, page 121) — 12 points

(a) — **3 points**

For the two rays that bound the full shadow: from each edge of the window past the board's edge on the same side (top edge of window past top edge of board, bottom past bottom), reaching the far wall 0.20 m on either side of the center line (1 grid square) — **1 point**

For the two rays that bound the partial shadows: from each edge of the window past the board's edge on the opposite side (crossing between the window and the board), reaching the far wall 1.40 m on either side of the center line (7 grid squares) — **1 point**

For labeling the far wall, in order, L, P, F, P, L, with the boundaries where the student's rays meet the wall — **1 point**



Courtesy figure. Black rays bound the full shadow (wall positions ±0.20 m); gray rays bound the partial shadows (wall positions ±1.40 m). Extra rays from points inside the window are fine if the boundary rays are present. A ray from the window's center (as for a point source) does not locate any boundary here.

Trap: Drawing only rays from the center of the window, or only the same-side rays, which gives one sharp shadow edge on each side and no partial shadow. The edges of an extended source must both be used.

(b) — **3 points**

For a correct similar-triangles (or slope) relationship for a boundary ray, for example: the ray from the top edge of the window (0.60 m above the center line) past the top edge of the board (0.40 m above) drops 0.20 m in 1.00 m, so it drops another 0.20 m in the next 1.00 m and meets the wall at 0.20 m above the center line — **1 point**

For the width of the full-shadow region, 0.40 m — **1 point**

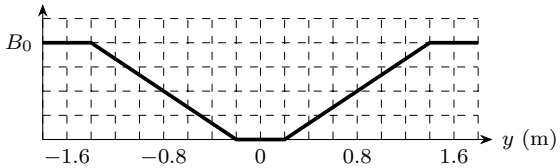
For the width of each partial-shadow region, 1.20 m (from 0.20 m to 1.40 m on each side; the opposite-edge ray from 0.60 m below the center line rises 1.00 m in 1.00 m, reaching 1.40 m above the center line) — **1 point**

General results: full-shadow width $= \frac{L}{a}w_b - \frac{L-a}{a}w_s$ *and partial-shadow width* $= \frac{L-a}{a}w_s$*, with window width* w_s*, board width* w_b*, board distance* a*, wall distance* L*. Students need not write these. Part (b) is scored on its own geometry; an error in the drawing for (a) is not held against (b).*

(c) — **2 points**

For brightness zero across the full-shadow region and B_0 across the fully lit regions, with boundaries consistent with (a) or (b) (correct: zero for $|y| < 0.20$ m, B_0 for $|y| > 1.40$ m) — **1 point**

For brightness increasing steadily (a straight line) from zero to B_0 across each partial-shadow region, symmetric about $y = 0$ — **1 point**



Courtesy plot. In a partial-shadow region the board's edge cuts the window at a point that moves steadily across the window as y changes, so the visible width (for example, 0.30 m at $y = 0.50$ m, 0.60 m at $y = 0.80$ m, 0.90 m at $y = 1.10$ m) is proportional to the distance from the edge of the full shadow. A ramp that is visibly curved but rises monotonically from zero to B_0 over the correct interval earns the second point; a step at a single position does not.

(d) **1 point**

For selecting "Less than" and correctly explaining that the two rays from opposite edges of the window that pass a given edge of the board spread apart after crossing there, so the width of the partial shadow grows with the distance from the board to the wall; either factor is enough: that distance is now smaller (0.50 m instead of 1.00 m), or the window subtends a smaller angle at the board's edge, so the rays spread less; the calculation 1.20 m $\times \frac{0.50}{1.50} = 0.40$ m also earns the point (no credit for this part without the correct selection) **1 point**

Trap: "Equal to, because the window and the board have not changed size." The partial-shadow width depends on the distances, not just on the widths; this is why a hand held close to a wall casts a sharp shadow.

(e) **2 points**

For locating the end of the region of no light by similar triangles: the ray from the top edge of the window (0.60 m) past the top edge of the strip (0.10 m) drops 0.50 m in 1.00 m and reaches the center line after a further 0.20 m, so the region ends 1.20 m from the window wall **1 point**

For concluding that the far wall (2.00 m away) has no full-shadow region because it is beyond 1.20 m, so every point on the far wall behind the strip receives light from part of the window (or: the same-side rays have crossed before reaching the wall, so the full-shadow width there would be negative, $\frac{2.00}{1.00}(0.20) - \frac{1.00}{1.00}(1.20) = -0.80$ m) **1 point**

The center of the far wall sees 0.80 m of the window (only the central 0.40 m is blocked). A correct "No" with no supporting geometry earns nothing.

Trap: Assuming any opaque object casts a full shadow on the wall. When the object is narrower than the source, the full shadow is a tapering region that can end before the wall (as the Moon's full shadow can end before reaching Earth).

(f) **1 point**

For selecting "Greater than" and correctly explaining that light now comes only from the center of the window, so a point on the wall is dark whenever the line from it to the center of the window passes through the board; those lines reach the wall 0.80 m on either side of the center line (the board's half-width doubled, since the wall is twice as far from the window as the board), a dark region 1.60 m wide, whereas before, points between 0.20 m and 0.80 m from the center line received light from the outer parts of the window (there is now no partial shadow, so the shadow is sharp) (no credit for this part without the correct selection) **1 point**

Trap: "Less than, because there is less light." Narrowing the source removes the light that reached the partial-shadow regions; the region with no light grows, and its edges become sharp.

Chapter 17a: Quantum physics

Question 17a.1 (LED turn-on voltages, page 124) — 12 points

(a) **3 points**

For the photon energy $E = hf$ **1 point**

For applying energy conservation per electron: $eV_{\text{on}} = hf$ **1 point**

For $V_{\text{on}} = \dfrac{h}{e}f$ **1 point**

Trap: Writing photon energy as depending on the brightness (or on the supply's current). In this model, each photon's energy is set by its frequency alone.

(b) **2 points**

For choosing frequency $f = c/\lambda$ (or $1/\lambda$) **1 point**

For correct values with units: $f \approx 4.76$, 4.96, 5.08, 5.71, 6.38 $\times 10^{14}$ Hz (or $1/\lambda$ in nm^{-1} or m^{-1}) **1 point**

(c) **3 points**

For correctly scaled and labeled axes (with units) that use more than half the grid **1 point**

For correctly plotted points **1 point**

For a reasonable best-fit line **1 point**

The axes need not start at zero.

(d) **2 points**

For using the slope of the best-fit line, about 4.0×10^{-15} V/Hz (not two data points far from the line), and relating it to h: slope $= h/e$ **1 point**

For $h \approx 6.4 \times 10^{-34}$ J·s (accept 6.0–7.2 $\times 10^{-34}$ J·s) **1 point**

A student who graphed against $1/\lambda$ uses slope $= hc/e$; that method earns full credit.

(e) **2 points**

For a prediction of about 3.1 V (accept 3.0–3.2 V, or a value consistent with the student's line) **1 point**

For "more energy per photon" with the correct reason: 400 nm is a shorter wavelength and higher frequency than 630 nm, and photon energy is proportional to frequency **1 point**

Question 17a.2 (Three-level atoms, page 126) 10 points

(a) **1 point**

For selecting the ranking $3 \to 2$ (longest), $2 \to 1$, $3 \to 1$ (shortest) and correctly justifying it: each photon's energy equals the difference between the two levels (1.0 eV, 3.0 eV, 4.0 eV), and wavelength is inversely proportional to photon energy, $\lambda = hc/E$ (no credit for this part without the correct ranking) **1 point**

(b) **2 points**

For $\lambda = \dfrac{hc}{E}$ with $E = 4.0$ eV **1 point**

For $\lambda \approx 310$ nm (3.1×10^{-7} m) **1 point**

(c)(i) **1 point**

For selecting "Yes" and correctly explaining that an atom absorbs a photon only if the photon's energy equals the difference between the atom's present level and a higher level, and the 3.0 eV photons of the $2 \to 1$ line match the gap from level 1, where essentially every atom in the cloud is, to level 2 (no credit for this part without the correct selection) **1 point**

(c)(ii) **1 point**

For selecting "No" and correctly explaining that the 1.0 eV photons of the $3 \to 2$ line match only the gap between levels 2 and 3, and almost no atoms in the cool cloud are in level 2; from level 1, 1.0 eV does not reach any higher level, so these photons pass through (no credit for this part without the correct selection) **1 point**

Trap: Answering "Yes" because "a gas absorbs the same wavelengths it emits." That is true only for transitions that start from a level the absorbing atoms are actually in.

(d) **1 point**

For checking all three lines and correctly explaining that atoms excited by absorbing lamp light then emit in all directions as they return to level 1: atoms in level 2 emit the $2 \to 1$ line, and atoms in level 3 return either directly ($3 \to 1$) or by way of level 2 ($3 \to 2$, then $2 \to 1$), so the $3 \to 2$ line appears even though the cloud does not absorb that line (no credit for this part without the correct selection) **1 point**

Trap: Checking only $3 \to 1$ and $2 \to 1$, on the idea that atoms re-emit only what they absorbed. An atom in level 3 can return to level 1 in two steps.

(e) **2 points**

For reasoning that an atom in level 1 can take from the electron only an amount of energy equal to a gap to a higher level, 3.0 eV (to level 2) or 4.0 eV (to level 3), and that 4.0 eV is more than the electron has, so the electron can excite the atom only to level 2 (a contrast with photon absorption, e.g., that the electron need not give up all its energy, is not required) **1 point**

For both possible final kinetic energies: 0.5 eV (atom excited to level 2) and 3.5 eV (no excitation) **1 point**

Trap: "No energy can be transferred, because 3.5 eV does not match any gap." That rule applies to absorbing a photon, which must give up all its energy; a colliding electron can give up part of its energy.

(f) **2 points**

For reasoning that ionizing an atom in level 1 requires at least 5.0 eV (from −5.0 eV to 0 eV), and that any photon with at least this energy can ionize it (the freed electron carries off the extra energy), so the longest wavelength corresponds to exactly 5.0 eV **1 point**

For $\lambda = \dfrac{hc}{E} = \dfrac{1240 \text{ eV} \cdot \text{nm}}{5.0 \text{ eV}} \approx 250 \text{ nm } (2.5 \times 10^{-7} \text{ m})$ **1 point**

Trap: Using 4.0 eV (the largest gap between bound levels), or claiming that only a photon of exactly 5.0 eV can ionize the atom.

Question 17a.3 (Rydberg atoms, page 129) 12 points

(a) **1 point**

For $Z = 1$ with the reasoning that the 36 inner electrons lie close to the nucleus, between it and the far-away outer electron (inside its orbit), so their charge, $-36e$, cancels all but $+e$ of the nucleus's charge, $+37e$: the outer electron is attracted by a net charge of $+e$, as the electron in hydrogen is **1 point**

Example response: $Z = 1$. The outer electron is far outside the inner shells, so all 36 inner electrons are between it and the nucleus. Their charge, $-36e$, cancels most of the nucleus's $+37e$, so the electron is attracted by a net charge of only $+e$, like the electron in hydrogen.

The language of shielding or screening is acceptable if the cancellation of 36 *of the* 37 *units of charge is clear. A statement that the charge enclosed by a sphere through the electron's orbit is* $+e$ *earns the point. A remark that the core acts like a point charge is welcome but not required. "*$Z = 1$ *because rubidium has one outer electron" does not earn the point.*

Trap: $Z = 37$, using the nuclear charge and ignoring the inner electrons.

(b) **3 points**

For finding the photon energy as the difference between the two levels, $\Delta E = (13.6 \text{ eV}) \left(\dfrac{1}{50^2} - \dfrac{1}{51^2} \right) \approx 2.11 \times 10^{-4} \text{ eV } (3.38 \times 10^{-23} \text{ J})$, using the value of Z from (a) **1 point**

For $f = \dfrac{\Delta E}{h} = \dfrac{2.11 \times 10^{-4} \text{ eV}}{4.14 \times 10^{-15} \text{ eV} \cdot \text{s}} \approx 5.1 \times 10^{10}$ Hz (accept 5.0–5.2 $\times 10^{10}$ Hz) **1 point**

For identifying the photon as microwave (wavelength $c/f \approx 5.9$ mm; "millimeter-wave" or "radio" is also accepted), consistent with the frequency found **1 point**

With $Z = 37$ *from (a),* $\Delta E \approx 0.289$ *eV,* $f \approx 7.0 \times 10^{13}$ *Hz, infrared; this earns all three points here. Computing each level separately (*-5.229×10^{-3} *eV and* -5.440×10^{-3} *eV) and subtracting requires carrying at least four figures; rounding each level to two figures (giving a difference of* 0.2×10^{-3} *eV,* $\approx 5 \times 10^{10}$ *Hz) is acceptable. For comparison, visible photons have energies of about* 2 *eV, roughly* 10^4 *times as great. Measured transitions between rubidium Rydberg levels with* n *near 50 do lie at tens of gigahertz; levels in which the electron never comes near the core match the model closely.*

Trap: Using only E_{51} or only E_{50} as the photon energy (about 5.3×10^{-3} eV, 1.3×10^{12} Hz).

(c) **2 points**

For $r_{50} = (0.0529 \text{ nm})(50)^2 \approx 132 \text{ nm } (1.3 \times 10^{-7} \text{ m})$, using the value of Z from (a) **1 point**

For reasoning that the orbit is about 40 times the spacing between molecules in air (diameter about 260 nm), so at ordinary pressure tens of thousands of other atoms or molecules would be inside the orbit (or would constantly collide with the atom), destroying the simple picture of an electron orbiting a $+e$ core and disturbing or removing the very weakly bound electron; at very low pressure the spacing between atoms is much larger than the orbit **1 point**

Example response: $r = (0.0529 \text{ nm})(2500) = 132$ nm. That is about 40 times the 3 nm spacing between molecules in air, so in air many molecules would sit inside the electron's orbit, and the electron would be constantly disturbed or knocked loose. In a very low-pressure gas, the atoms are much farther apart than the size of the orbit, so each Rydberg atom is undisturbed.

Also acceptable for the second point: the electron's binding energy, $|E_{50}| \approx 0.005$ eV, is much less than the typical kinetic energy of molecules at room temperature, $\frac{3}{2}k_BT \approx 0.04$ eV, so frequent collisions would ionize the atom; at low pressure collisions are rare. With $Z = 37$ from (a), $r \approx 3.6$ nm, comparable to the spacing; a consistent argument earns the second point.

(d) **3 points**

For $\Delta E = (13.6 \text{ eV})\left(\frac{1}{100^2} - \frac{1}{101^2}\right) \approx 2.68 \times 10^{-5}$ eV (or $f \approx 6.5 \times 10^9$ Hz) **1 point**

For comparing with (b): this photon has about $\frac{1}{8}$ of the energy ($2.11 \times 10^{-4}/2.68 \times 10^{-5} \approx 7.9$), not $\frac{1}{4}$, so the student's claim is incorrect **1 point**

For explaining the flaw: the photon's energy is the *difference* between two neighboring levels, not a level's energy. Doubling n makes each level's energy $\frac{1}{4}$ as large, but it also makes neighboring levels closer to each other relative to their size (n and $n+1$ differ by about $\frac{1}{50}$ of n at $n = 50$ but only $\frac{1}{100}$ at $n = 100$), so the spacing shrinks by about another factor of 2 **1 point**

Example response: $\Delta E = 13.6(1/100^2 - 1/101^2) \approx 2.7 \times 10^{-5}$ eV, about $\frac{1}{8}$ of the 2.1×10^{-4} eV in (b). The student is wrong. The photon's energy is the gap between two levels. When n doubles, each level becomes $\frac{1}{4}$ as deep, but going from n to $n+1$ is also a smaller fractional change in n (1 part in 100 instead of 1 in 50), so the two levels differ by a smaller fraction of their size. The gap shrinks by about $4 \times 2 = 8$.

For large n, $\Delta E \approx 2(13.6 \text{ eV})/n^3$; students are not expected to derive this, and an explanation in words as above earns the third point. "The student forgot the square" or a bare ratio of 8 without reasoning about the gap does not earn the third point. Another valid route earns the second and third points: doubling every n turns the $51 \to 50$ transition into $102 \to 100$, whose energy is $\frac{1}{4}$ as large; but $101 \to 100$ spans only one of those two steps, about half that gap, so it has about $\frac{1}{8}$ of the energy. With $Z = 37$ carried from (a), both photon energies are multiplied by 1369 and the ratio is unchanged.

Trap: Agreeing with the student by comparing E_{100} with E_{50} (ratio 4) instead of comparing the gaps.

(e) **3 points**

For $E_5 = -\frac{13.6 \text{ eV}}{5^2} \approx -0.544$ eV, with the observation that the measured energy is about 8 times as negative: the real electron is much more tightly bound than the model predicts **1 point**

For identifying the failed assumption: that all 36 inner electrons are always between the outer electron and the nucleus (that the electron stays far outside the inner shells), so that it feels a net charge of only $+e$ **1 point**

For explaining the direction: when the outer electron is among the inner electrons, fewer than 36 of them are between it and the nucleus, so the net charge attracting it is more than $+e$; the stronger attraction binds it more tightly, so its energy is more negative than the model's **1 point**

Example response: The model gives $E_5 = -13.6/25 = -0.544$ eV, but the measured energy is -4.18 eV, so the real electron is bound about 8 times as tightly. Part (a) assumed that all 36 inner electrons are between the outer electron and the nucleus, so that it feels a net charge of only $+e$. At $n = 5$ the electron spends part of its time among the inner electrons, where fewer than 36 of them are between it and the nucleus. There it is attracted by more than $+e$, so it is held more tightly and its energy is lower.

An effective Z greater than 1 (about 2.8 if $E = -13.6Z^2/n^2$ is forced to fit) stated with this reasoning is equally good. The third point requires the link from "fewer inner electrons between" to "greater net attracting charge" to "more tightly bound"; repeating the stem's sentence about the electron's location does not earn it. With $Z = 37$ from (a), the model gives -745 eV; a consistent answer (the assumption that the full nuclear charge acts fails, because the inner electrons cancel most of it, so the real electron is bound far more weakly) earns all three points.

Trap: "The measured energy is lower because the electron is closer, so the $1/n^2$ formula should give it anyway." The formula already includes the smaller orbit; the discrepancy comes from the net charge acting on the electron.

Chapter 17b: Subatomic physics

Question 17b.1 (Alpha decay of polonium-210, page 132) **10 points**

(a) **2 points**

For a mass number of 206, explained by conservation of nucleon number: the alpha particle carries away 4 nucleons, $210 - 4 = 206$ **1 point**

For an atomic number of 82, explained by conservation of charge: the alpha particle carries away 2 protons, $84 - 2 = 82$, so the element is lead: ${}^{206}_{82}\mathrm{Pb}$ **1 point**

A correct decay equation, ${}^{210}_{84}\mathrm{Po} \rightarrow {}^{206}_{82}\mathrm{Pb} + {}^{4}_{2}\mathrm{He}$*, with a statement that the top numbers and the bottom numbers balance, earns both points. Each number needs its reason; a bare symbol earns nothing.*

Trap: Choosing lead-210 or bismuth-210 from the table because the mass "stays about 210." The alpha particle takes 4 nucleons with it. Choosing mercury-206 or thallium-206 gets the nucleon number right but not the charge.

(b) **2 points**

For finding the mass difference between the polonium-210 atom and the products, using the atomic masses from the table: $\Delta m = 209.982874\ \mathrm{u} - (205.974465\ \mathrm{u} + 4.002603\ \mathrm{u}) = 0.005806\ \mathrm{u}$ **1 point**

For converting with $E = \Delta m\, c^2$: $E = (0.005806)(931.5\ \mathrm{MeV}) \approx 5.41\ \mathrm{MeV}$ (accept 5.4 MeV) **1 point**

Atomic masses include electrons: 84 on the left, $82 + 2$ *on the right, so the electron masses cancel and no correction is needed. A student who converts to kilograms and joules (*$\Delta m \approx 9.64 \times 10^{-30}$ *kg,* $E \approx 8.67 \times 10^{-13}$ *J* ≈ 5.41 *MeV) earns full credit. A student who used a wrong daughter from (a) earns the first point for a correctly set-up mass difference with that isotope (a negative result should be recognized as impossible for a decay that occurs).*

Trap: Subtracting mass numbers ($210 - 206 - 4 = 0$) and concluding that no energy is released, or rounding the masses before subtracting so that the difference disappears.

(c) **3 points**

For reasoning that the polonium nucleus is initially at rest, so the system's total momentum is zero, and no external force acts during the decay, so the total momentum remains zero: the alpha particle and the lead nucleus have momenta of equal magnitude in opposite directions **1 point**

For reasoning that with equal momentum magnitudes, the less massive alpha particle has the greater kinetic energy, using $K = \dfrac{p^2}{2m}$, or $K = \frac{1}{2}pv$ with the same p and the alpha's greater speed, or equivalently $m_\alpha v_\alpha = m_{\mathrm{Pb}} v_{\mathrm{Pb}}$ so the alpha's speed is about 51 times as great, and $K = \frac{1}{2}mv^2$ depends on the square of the speed but only on the first power of the mass **1 point**

For a logical, relevant paragraph that reaches "greater than" and contains no incorrect or contradictory statements **1 point**

The answer blanks are not scored. "The alpha is lighter, so it moves faster" without the momentum argument does not earn the first point; "it moves faster, so it has more kinetic energy" does not earn the second point, because the lead nucleus is more massive.

Example response: Take the polonium nucleus as the system. It starts at rest, so its momentum is zero, and during the decay only internal forces act between the alpha particle and the lead nucleus, so the total momentum stays zero. The two pieces must therefore move in opposite directions with momenta of equal magnitude p. Kinetic energy can be written $K = \frac{p^2}{2m}$. With the same p, the piece with the smaller mass has the larger kinetic energy. The alpha particle's mass is about $\frac{4}{206}$ of the lead nucleus's mass, so the alpha particle's kinetic energy is about 51 times that of the lead nucleus.

Trap: "The same as, because the forces on the two pieces are equal and opposite." Equal forces for the same time give equal momenta, not equal kinetic energies. Another trap is "the energy is split equally," or a claim that the lead nucleus gets more energy because it is bigger.

(d) **2 points**

For using momentum conservation together with energy conservation, $K_\alpha + K_{\mathrm{Pb}} = E$ from (b) and $\dfrac{K_{\mathrm{Pb}}}{K_\alpha} = \dfrac{m_\alpha}{m_{\mathrm{Pb}}}$, or an equivalent route (e.g., $K_\alpha = E\,\dfrac{m_{\mathrm{Pb}}}{m_{\mathrm{Pb}} + m_\alpha}$) **1 point**

For $K_\alpha \approx 5.31$ MeV (accept 5.30–5.31 MeV, or a value consistent with the student's (b): about 98% of it) **1 point**

Mass numbers (206 and 4) or the atomic masses from the table may be used for the mass ratio; the result is the same to three significant figures. The lead nucleus gets about 0.10 *MeV. Solving for speeds first and then kinetic energies is acceptable but slow; full credit if correct.*

Trap: Answering 5.41 MeV, giving all the released energy to the alpha particle, or splitting it equally (2.7 MeV each), which violates momentum conservation.

(e) **1 point**

For selecting "Bismuth-210" and correctly explaining that in beta-minus decay the nucleon number is unchanged (210) and the atomic number increases by 1, either because a neutron becomes a proton and an electron is emitted, or by conservation of charge and nucleon number with the emitted electron's charge -1 and nucleon number 0; the parent therefore has $Z = 84 - 1 = 83$ and $A = 210$ (no credit for this part without the correct selection) **1 point**

Trap: Choosing astatine-210 by thinking the emitted electron's negative charge must come from a nucleus with more protons, i.e., that beta-minus decay lowers Z. Choosing lead-210 counts the change in Z as 2.

Question 17b.2 (Technetium-99m vial, page 135) 12 points

(a) **2 points**

For a balanced equation with the electron written in consistent notation: ${}^{99}_{42}\mathrm{Mo} \rightarrow {}^{99}_{43}\mathrm{Tc} + {}^{0}_{-1}e\ (+\ \bar{\nu})$ (or β^- with charge -1 and nucleon number 0 shown) **1 point**

For explaining both numbers: the electron carries no nucleons, so conservation of nucleon number keeps $A = 99$; the electron carries charge -1, so conservation of charge requires the nucleus's charge to rise from $+42$ to $+43$ (a neutron becomes a proton), which makes the element technetium **1 point**

The antineutrino is correct but not required. Because the table gives Z for technetium and the name gives 99, the numbers alone show nothing; the second point requires the conservation reasoning for both. A positron (${}^{0}_{+1}e$) with $Z = 41$ (niobium) earns neither point.

Trap: Writing the electron as ${}^{0}_{+1}$ or ${}^{1}_{-1}$, or arguing that emitting negative charge must remove a proton.

(b) **1 point**

For selecting "Same Z, same A" and correctly explaining that a gamma-ray photon has no charge and is not a nucleon, so by conservation of charge and of nucleon number the atomic number and mass number are unchanged (the nucleus is still ${}^{99}_{43}\mathrm{Tc}$); a statement that the nucleus only loses energy, moving to a lower-energy state, is optional and not sufficient by itself (no credit for this part without the correct selection) **1 point**

Trap: Treating gamma emission as changing the element, by analogy with alpha or beta decay.

(c) **1 point**

For a column labeled as the net (or sample) count rate in counts/min with the background subtracted from every entry: 950, 694, 506, 332, 246, 147, 123, 95, 59 **1 point**

Arithmetic slips in one value are tolerated if the label and method are clear and the other values are correct.

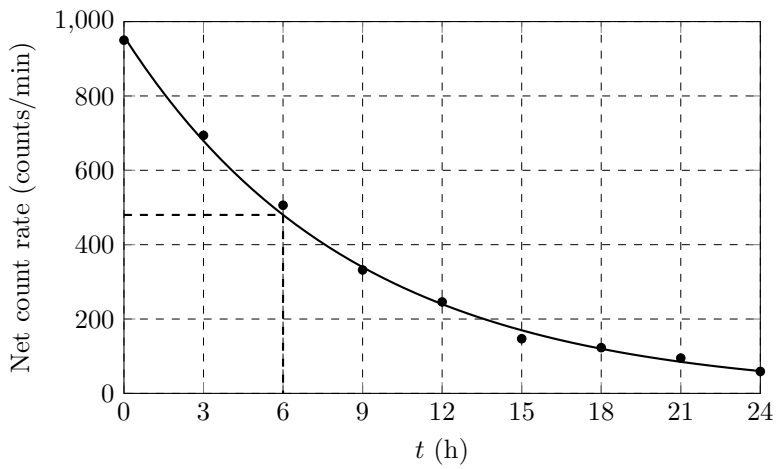
(d) **3 points**

For axes labeled with quantities and units (time in h; net count rate in counts/min) and scaled linearly so that the data use more than half the grid in each direction **1 point**

For correctly plotting the net count rates from (c) (or the student's values from (c)) **1 point**

For a smooth curve that decreases with decreasing steepness and passes among the points, approaching zero rather than leveling off at a nonzero value (a curve through every point earns this point only if it is smooth) **1 point**

A student who plots the raw count rates can earn the first point only. A student who plots ln *of the net rate with a straight line earns full credit if it is labeled and scaled correctly.*



Courtesy figure: the curve is $960 \cdot 2^{-t/6.0\ h}$ *counts/min; the dashed lines show one halving, from about 960 to 480 counts/min in about 6 h. Points attach to the features described in words.*

(e) **2 points**

For a method shown on the graph: reading the time for the curve to fall from any value to half that value (for example from about 960 to 480 counts/min), or the time between successive halvings **1 point**

For a half-life of about 6 h (accept 5.5–6.5 h) and a conclusion that it is consistent with 6.0 h, given the random scatter of the counts (or a conclusion consistent with the student's value) **1 point**

A best fit of the data gives 6.0 *h. Halving times read from single data points rather than the curve vary from about 5 h to 6.5 h because of scatter; a student who reports such a value earns the method point and the second point if the value falls in the accepted range.*

Trap: Reading the time at which the rate reaches half of the *raw* first value, 535 counts/min, which gives about 7.5 h.

(f) **1 point**

For selecting "No" and correctly explaining that the recorded rates include the 120 counts/min background, which does not decay; the sample's own rate must be found by subtracting it, and the sample's rate does fall to half in about 6 h, consistent with Tc-99m (the raw rate levels off near 120 counts/min instead of approaching zero, which is why the raw halving time is too long) (no credit for this part without the correct selection) **1 point**

Trap: "Yes, because the measured half-life is longer than 6.0 h." The longer halving time is produced by the constant background, not by a second isotope.

(g) **2 points**

For reasoning that 12 h is two half-lives, so the activity falls by a factor of $2^2 = 4$ between preparation and injection (or for $A = A_0 \left(\frac{1}{2}\right)^{t/T_{1/2}}$ solved for A_0) **1 point**

For an activity at preparation of $4 \times 740 = 2960$ MBq ($\approx 3.0 \times 10^3$ MBq, or 3.0×10^9 decays per second) **1 point**

Trap: Dividing by 4 (185 MBq), which finds the activity 12 h *after* injection rather than before it; or using 2 half-lives as a factor of 2 (1480 MBq).

Chapter 18a: Ideal gases and kinetic theory

Question 18a.1 (Helium and xenon, page 139) 10 points

(a) **1 point**

For selecting "The same as" and correctly justifying it: both gases are at the same temperature (thermal equilibrium; collisions share energy until the averages match), and the average translational kinetic energy depends only on temperature, $\bar{K} = \frac{3}{2}k_B T$, not on mass (no credit for this part without the correct selection) **1 point**

Trap: "Less than: xenon is heavier, so it has more kinetic energy." At the same temperature the heavier atoms simply move more slowly.

(b) **3 points**

For setting the average kinetic energies equal: $\frac{1}{2}m_{\text{He}}v_{\text{He}}^2 = \frac{1}{2}m_{\text{Xe}}v_{\text{Xe}}^2$ (or using $v_{\text{rms}} = \sqrt{3k_BT/m}$) **1 point**

For $\dfrac{v_{\text{He}}}{v_{\text{Xe}}} = \sqrt{\dfrac{m_{\text{Xe}}}{m_{\text{He}}}}$ **1 point**

For $\sqrt{33} \approx 5.7$ **1 point**

(c) **2 points**

For reasoning that the change in momentum when an atom bounces off a wall is $2mv$, and with equal average kinetic energies mv^2 is the same for both, so $mv \propto \sqrt{m}$: a helium atom's change in momentum is smaller, by a factor of $\sqrt{33} \approx 5.7$, even though it is faster **1 point**

For reasoning that an atom strikes the walls at a rate proportional to its speed (the time between hits on a given wall is the round-trip distance divided by the speed), so a helium atom strikes the walls more often, about 5.7 times as often, as found from (b) **1 point**

The factor $\sqrt{33}$ may be stated as "the speed ratio from (b)"; a correct comparison without the numerical factor earns the points if the reasoning shows the dependence on m and v.

(d) **1 point**

For selecting "The same as" and correctly explaining that the pressure from one kind of atom depends on the momentum change per hit times the number of hits per second; from (c), helium's smaller momentum change per hit is exactly offset by its more frequent hits (the product is proportional to mv^2, the same for both), and there are equal numbers of each kind of atom in the same volume, so the contributions are equal (consistent with $P_i = N_ik_BT/V$) (no credit for this part without the correct selection) **1 point**

Trap: "Greater than, because helium atoms are faster," or "less than, because xenon atoms hit harder." Each argument counts only one of the two effects.

(e) **2 points**

For applying $PV = Nk_BT$ (or $PV = nRT$ and Avogadro's number) with the volume in m^3 and the absolute temperature, 293 K, either to the whole mixture (then taking half of N, since the numbers are equal) or to the helium alone with its share of the pressure, 5.0×10^4 Pa **1 point**

For $N_{\text{He}} \approx 6.2 \times 10^{21}$ **1 point**

Trap: Using the total pressure for helium alone (giving 1.2×10^{22}), or using 20 in place of 293 K.

(f) **1 point**

For selecting "Decrease" and correctly explaining that atoms escape when they happen to reach the pinhole, and helium atoms, being faster, reach the hole more often (as in (c)), so helium escapes at a greater rate than xenon and the ratio of helium to xenon atoms in the bulb decreases (no credit for this part without the correct selection) **1 point**

Trap: "Stay the same, because the two gases have equal partial pressures." Equal pressures mean equal momentum delivered per second, not equal numbers of atoms reaching the hole per second.

Question 18a.2 (Syringe in water baths, page 141) 12 points

(a) **2 points**

For one valid step, for example waiting until the trapped air reaches the bath temperature (the volume stops changing) before reading **1 point**

For a second valid step, for example keeping the whole air sample under water, stirring the bath and measuring its temperature near the syringe, tapping the plunger to make sure it is not stuck, reading the scale at eye level, or repeating readings **1 point**

(b) **2 points**

For using $PV = nRT_K$ with P and n constant, so V is proportional to the absolute temperature T_K **1 point**

For reasoning that $T_K = T_{\text{C}} + 273$, so V is a linear function of T_{C} (straight, but not through $T_{\text{C}} = 0$) **1 point**

(c) **3 points**

For axes labeled with quantities and units, scaled so that the horizontal axis reaches about $-300\,^\circ\text{C}$ **1 point**

For correctly plotted points **1 point**

For a best-fit line extended to $V = 0$ **1 point**

Trap: Starting the temperature axis at 0 °C, which puts the zero of the plot at the freezing point of water and makes it look as if V should be proportional to T in Celsius.

(d) **3 points**

For an intercept read from the student's own extended best-fit line, with the point where the line reaches $V = 0$ marked on the graph (a bare number with no marked intercept or stated line does not earn this point) **1 point**

For a value of about $-270\,°\mathrm{C}$ (accept $-240\,°\mathrm{C}$ to $-300\,°\mathrm{C}$, or a value consistent with the student's line), matching the marked intercept **1 point**

For explaining with the ideal gas law that $V = nRT_K/P$ is zero only when $T_K = 0$, so this temperature is absolute zero **1 point**

A value of $-273\,°C$ quoted from memory, with no line drawn to $V = 0$, earns only the third point.

(e) **1 point**

For selecting "The same as" and correctly justifying it: doubling the amount of air doubles every volume (and the slope), but by $PV = nRT_K$ the volume is still proportional to the absolute temperature, so the line still reaches $V = 0$ at $T_K = 0$; the intercept does not depend on n (no credit for this part without the correct selection) **1 point**

(f) **1 point**

For using $PV = nRT$ with the temperature in kelvins, either with one data point or with the slope of the best-fit line ($\Delta V/\Delta T = nR/P$, about 0.074 mL/K), to get $n \approx 8.8 \times 10^{-4}$ mol (accept 8.5–9.3 $\times 10^{-4}$ mol) **1 point**

Trap: Using the temperature in degrees Celsius, which gives a different and wrong value for each data point.

Question 18a.3 (Two-peaked speed distribution, page 143) 12 points

(a) **1 point**

For selecting "The same as" and correctly justifying it: the two gases are at the same temperature (thermal equilibrium; collisions share energy until the averages match), and the average translational kinetic energy of an atom depends only on the temperature, $\bar{K} = \frac{3}{2}k_BT$, not on its mass (no credit for this part without the correct selection) **1 point**

Trap: "Less than: krypton atoms are heavier, so they have more kinetic energy." At the same temperature the heavier atoms move more slowly.

(b) **1 point**

For selecting "The right peak" and correctly justifying it: helium and krypton atoms have the same average kinetic energy, $\frac{1}{2}mv^2$, so the lighter helium atoms must have the greater typical speeds (or $v_{\mathrm{rms}} = \sqrt{3k_BT/m}$ with the same T for both gases, so the smaller mass gives the greater speed), and the right peak is at the greater speed (no credit for this part without the correct selection) **1 point**

An answer consistent with an incorrect "greater than" in (a) (e.g., "helium has more kinetic energy and less mass, so it is faster") does not earn the point: the reasoning includes an incorrect statement.

Trap: "The left peak, because the taller peak belongs to the lighter, more numerous, or more energetic atoms." The position of a peak, not its height, shows the typical speed.

(c) **3 points**

For reasoning that the average kinetic energies are equal, $\frac{1}{2}m_{\mathrm{He}}v_{\mathrm{rms,He}}^2 = \frac{1}{2}m_{\mathrm{Kr}}v_{\mathrm{rms,Kr}}^2$ (or $v_{\mathrm{rms}} = \sqrt{3k_BT/m}$ with the same T), and that the peak speeds have the same ratio as the rms speeds **1 point**

For the predicted ratio $\dfrac{v_{\mathrm{He}}}{v_{\mathrm{Kr}}} = \sqrt{\dfrac{m_{\mathrm{Kr}}}{m_{\mathrm{He}}}} = \sqrt{21} \approx 4.6$ **1 point**

For reading the peak speeds from the graph (about 1100 m/s and 250 m/s; accept 1050–1200 m/s and 220–280 m/s), a ratio of about 4.4 (any ratio consistent with the values read, roughly 3.7–5.5), and a statement that this agrees with the prediction within the precision of reading the graph **1 point**

The curve's peaks are at 1117 *m/s and* 249 *m/s (ratio* 4.5*). The krypton peak is pulled very slightly to the right by the small number of slow helium atoms, which is why the graph's ratio is a little below* $\sqrt{21}$*; students are not expected to notice this. For each gas alone, the peak is at* $\sqrt{2k_BT/m}$*:* 1117 *m/s and* 244 *m/s.*

Trap: Inverting the ratio ($\sqrt{m_{\text{He}}/m_{\text{Kr}}} \approx 0.22$) or omitting the square root (ratio 21).

(d)(i) **2 points**

For reasoning that the number of atoms of one kind (here, the left-peak atoms, krypton, from (b)) is represented by the area under that gas's part of the curve, not by the height of its peak **1 point**

For reasoning that the height of a peak also depends on how spread out the speeds are: the krypton atoms' speeds are crowded into a narrow range (roughly 0 to 600 m/s), so the krypton hump is tall and narrow, while the helium atoms' speeds are spread over a much wider range (roughly 400 to 2500 m/s), so the helium hump is short and wide and can have as much area or more **1 point**

Example response: The fraction of atoms that are krypton is the area under the krypton hump, not its height. The krypton atoms are slow and their speeds lie in a narrow range, so their hump is tall but narrow. The helium speeds are spread over a range several times as wide, so the helium hump is short, but its area can still be larger. A taller peak does not mean more atoms.

Trap: Agreeing with the student, or saying the taller peak means the krypton is at a higher temperature. Both gases are at the same temperature; height reflects both the number of atoms and the spread of their speeds.

(d)(ii) **1 point**

For selecting "More right-peak atoms" (helium) and correctly explaining, using the graph, that the right (helium) hump has the greater area: for example, if the numbers were equal, the helium hump would be the krypton hump stretched horizontally by the factor 4.6 from (c) and so 4.6 times shorter, but the krypton peak is only about 3 times as tall, so the helium hump has more area; or an area estimate (helium hump about 4.5×10^{-4} s/m high and roughly 1300 m/s wide, area about 0.6; krypton hump about 14×10^{-4} s/m high and roughly 300 m/s wide, area about 0.4) (no credit for this part without the correct selection) **1 point**

The graph was computed for 60% *helium and* 40% *krypton. Rough area estimates (counting grid squares, or height times width at half height) are acceptable if they clearly favor the right hump; a statement such as "the left hump has about* 0.4 *of the area, so the rest, about* 0.6*, is under the right hump" earns the point; with the scaling argument, the ratio* $N_{He}/N_{Kr} \approx 4.6/3.2 \approx 1.4$ *(actual* 1.5*) is not required. "More right-peak atoms because the right hump is wider" without comparing area (height and width together) does not earn the point.*

(e) **2 points**

For $v_{\text{rms}} = \sqrt{\dfrac{3k_BT}{m}} = \sqrt{\dfrac{3(1.38 \times 10^{-23}\ \text{J/K})(300\ \text{K})}{6.65 \times 10^{-27}\ \text{kg}}} \approx 1.37 \times 10^3$ m/s (or $\frac{3}{2}k_BT = \frac{1}{2}mv_{\text{rms}}^2$ solved for v_{rms}) **1 point**

For explaining that the helium hump is not symmetric: it has a long tail toward high speeds, and in the root-mean-square average the speeds are squared, so the few very fast atoms raise the average well above the most common speed (the peak); the rms speed is therefore greater than the speed at the peak, about 1100 m/s **1 point**

Example response: $v_{\text{rms}} = \sqrt{3(1.38 \times 10^{-23})(300)/(6.65 \times 10^{-27})} \approx 1370$ m/s. The peak is at the most common speed, about 1100 m/s. The helium curve extends much farther above the peak (to about 2500 m/s) than below it (it must stop at zero speed), and squaring the speeds makes the fast atoms count even more, so the rms speed lies to the right of the peak.

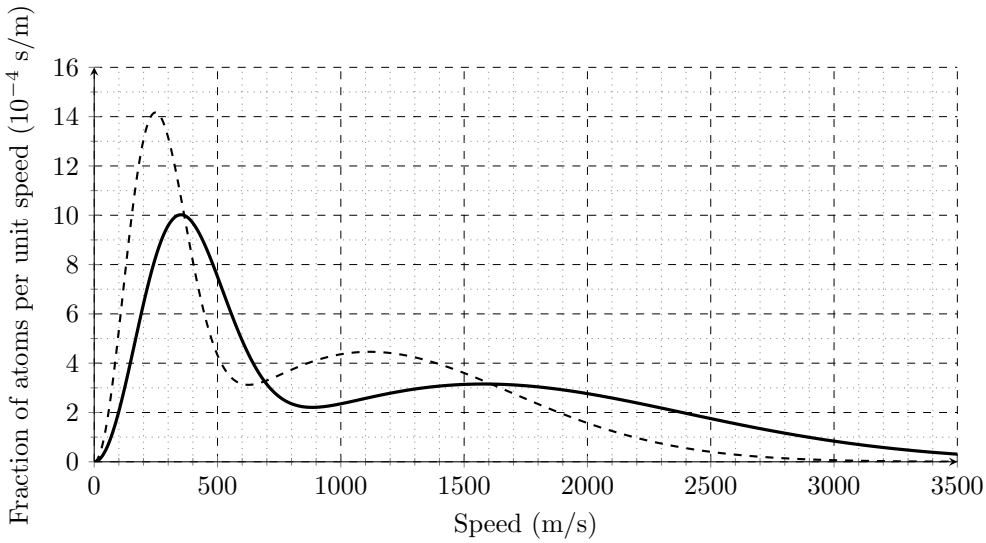
An explanation using only the asymmetry (the average of a distribution with a long high-speed tail lies above its peak) earns the second point. The average speed, 1260 *m/s, also lies above the peak; mentioning it is not required.*

Trap: Concluding that the graph or the calculation is wrong, or using T in °C.

(f) **2 points**

For a sketch in which both peaks are at higher speeds, by roughly the factor $\sqrt{2} \approx 1.4$ (helium peak near 1600 m/s, accept 1350–1850; krypton peak near 350 m/s, accept 290–420) **1 point**

For a sketch in which both peaks are lower and both humps are wider, so that each hump keeps about the same area (the krypton peak still clearly taller than the helium peak, and the curve still starting at zero speed) **1 point**



Courtesy figure: the original 300 K curve (dashed) and the 600 K curve (solid). At 600 K the peaks are at about 1580 *m/s and* 350 *m/s, with heights about* 3.2×10^{-4} *and* 10×10^{-4} *s/m. Students may get the factor from (c): every characteristic speed is proportional to* $v_{rms} \propto \sqrt{T}$*. A sketch that doubles the peak speeds (treating speed as proportional to* T*) does not earn the first point. The second point requires both features (lower and wider); a sketch with taller or unchanged-height peaks does not earn it.*

Trap: Moving the peaks right without lowering them, which adds atoms (area), or moving only the helium peak.

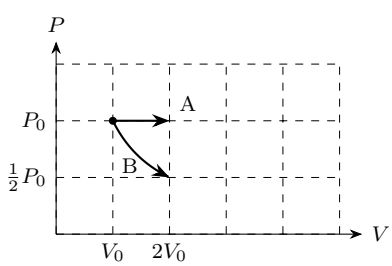
Chapter 18b: Thermodynamics: first law

Question 18b.1 (Two gas expansions, page 146) — 10 points

(a) — **2 points**

For process A drawn as a horizontal line at P_0 from V_0 to $2V_0$ — **1 point**

For process B drawn as a curve, concave up, from the initial point down to $\left(2V_0, \frac{1}{2}P_0\right)$ — **1 point**



Courtesy plot.

(b) — **1 point**

For $2T_0$, justified by the ideal gas law: at constant pressure with a fixed amount of gas, V is proportional to T, so doubling the volume doubles the temperature (no credit for $2T_0$ without the justification) — **1 point**

(c) — **1 point**

For explaining that the work done by the gas equals the area under its curve on the P–V graph between V_0 and $2V_0$, and that this area is larger for A because A stays at P_0 while B's pressure falls below P_0 throughout the expansion — **1 point**

(d) — **2 points**

For explaining that the internal energy of an ideal gas depends only on its temperature, so with the temperature constant the internal energy does not change during B — **1 point**

For explaining that the gas expands and so does positive work on the piston, and by the first law of thermodynamics, with no change in internal energy, the energy the gas gives up as work must be supplied to it by heating, in an amount equal to that work **1 point**

Trap: "The temperature stays the same, so no heat flows." Constant temperature means no change in internal energy, not zero heating; the water bath keeps supplying the energy that the gas spends pushing the piston out.

(e) **1 point**

For selecting "Process A" and correctly explaining, with the first law, that in A the heating must supply both the increase in internal energy (the temperature rises) and the work done by the gas, which is the larger work, while in B the heating supplies only the smaller work and the internal energy does not change (no credit for this part without the correct selection) **1 point**

Trap: "The same, because both end at volume $2V_0$." The end states differ (pressure P_0 versus $\frac{1}{2}P_0$), and heat depends on the path, not just the final volume.

(f) **3 points**

For the work done by the gas, $P_0\,\Delta V = P_0V_0$ (or work done on the gas, $-P_0V_0$) **1 point**

For the change in internal energy, $\Delta U = \frac{3}{2}nR\,\Delta T = \frac{3}{2}\,\Delta(PV) = \frac{3}{2}P_0V_0$ **1 point**

For $Q = \Delta U + W_{\text{by gas}} = \frac{5}{2}P_0V_0$ **1 point**

Trap: A sign error with the first law gives $Q = \frac{1}{2}P_0V_0$; earns 2 of 3. Using $\Delta U = \frac{3}{2}nR\,\Delta T$ is fine, but the answer must be rewritten in P_0 and V_0 using $nR\,\Delta T = P_0\,\Delta V$.

Question 18b.3 (Rectangular cycle, page 148) 12 points

(a) **2 points**

For using the area enclosed by the cycle (or the sum of the work in each process: $+4P_0V_0$ for A→B, 0, $-2P_0V_0$ for C→D, 0) **1 point**

For $W_{\text{by gas}} = 2P_0V_0$ (positive, since the cycle runs clockwise) **1 point**

(b)(i) **2 points**

For identifying the correct aspect: the internal energy does not change over the cycle **1 point**

For a correct reason: internal energy depends only on the state (for an ideal gas, on temperature, $U = \frac{3}{2}nRT = \frac{3}{2}PV$), and the gas returns to the same state **1 point**

(b)(ii) **2 points**

For identifying the incorrect aspect: the conclusion that the net heating is zero **1 point**

For explaining with the first law, $\Delta U = Q + W_{\text{on gas}} = Q - W_{\text{by gas}}$: with $\Delta U = 0$, the net heat added equals the net work done by the gas, $2P_0V_0$, which is not zero **1 point**

Trap: Agreeing with the student. Heat, like work, depends on the path, not only on the starting and ending states.

(c) **3 points**

For the change in internal energy: $\Delta U = \frac{3}{2}\Delta(PV) = \frac{3}{2}(6P_0V_0 - 2P_0V_0) = 6P_0V_0$ **1 point**

For the work done by the gas: $2P_0 \cdot 2V_0 = 4P_0V_0$ **1 point**

For $Q = \Delta U + W_{\text{by gas}} = 10P_0V_0$ **1 point**

Trap: Writing $Q = 4P_0V_0$ (the work alone) or $6P_0V_0$ (the internal-energy change alone).

(d) **2 points**

For reasoning that the volume is constant, so the gas does no work and $Q = \Delta U = \frac{3}{2}\Delta(PV) = \frac{3}{2}V\Delta P$ **1 point**

For $Q = \frac{3}{2}(3V_0)(P_0 - 2P_0) = -\frac{9}{2}P_0V_0$: $\frac{9}{2}P_0V_0$ is transferred out of the gas **1 point**

Trap: Writing $Q = 0$ because no work is done. With no work, all of the change in internal energy comes from heating.

(e) **1 point**

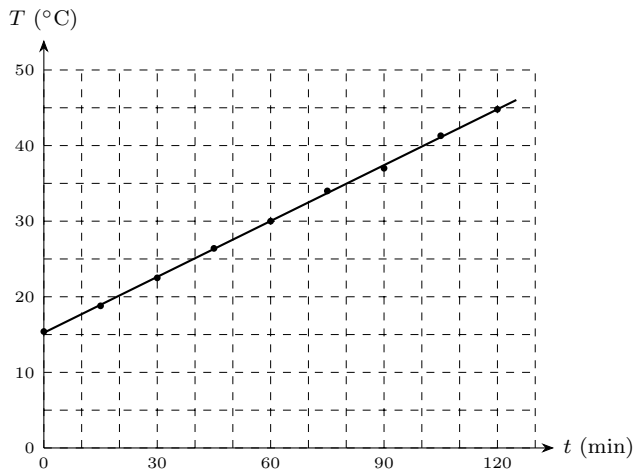
For selecting the ranking $T_B > T_C > T_A > T_D$ and correctly justifying it: for an ideal gas with fixed n, $T \propto PV$, and the products PV are 2, 6, 3, 1 (in units of P_0V_0) for A, B, C, D (no credit for this part without the correct ranking) **1 point**

Question 18b.4 (Heat-pump water heater, page 150) 12 points

(a) **2 points**

For axes labeled with quantities and units (for example, t in min or s, T in °C), with linear scales that use at least half of the grid in each direction, and the nine points plotted correctly **1 point**

For a single straight best-fit line with points on both sides of it (not a dot-to-dot line, and not forced through the origin) **1 point**



Courtesy plot; the line shown is the least-squares line, slope 0.246 *°C/min.*

(b) **2 points**

For a slope found from two points on the best-fit line (not two data points unless they lie on the line) or from a linear regression of the data, about 0.25 °C/min = 4.1×10^{-3} °C/s (accept 0.23–0.26 °C/min) **1 point**

For using $Q = mc\,\Delta T$ so that the rate is $mc\,(\Delta T/\Delta t)$: (150 kg)(4190 J/(kg·°C))(4.1×10^{-3} °C/s) $\approx$ 2.6×10^3 W (accept 2.4–2.7 kW, consistent with the student's slope) **1 point**

Trap: Dividing one temperature reading by its time (for example 44.8/120) instead of using the slope, which treats the water as if it started at 0 °C.

(c) **3 points**

For reasoning that the refrigerant runs around the same loop over and over and, in steady operation, returns to the same state, so over any whole number of trips (or over any time interval in steady operation) its internal energy does not change: $\Delta U = 0$ **1 point**

For applying the first law, $\Delta U = Q + W$, to the refrigerant: (energy in from the air) + (work done by the compressor) − (energy given to the water) = 0, so the rate of energy taken from the air is the rate of energy given to the water minus the electrical power **1 point**

For 2.6 kW − 1.00 kW $\approx$ 1.6 kW (accept a value consistent with part (b)) **1 point**

Trap: Writing that the refrigerant "uses up" the electrical energy, or adding the electrical power to the rate into the water (3.6 kW). The water's gain is the sum of the two inputs to the refrigerant, not the difference.

(d)(i) **1 point**

For identifying a correct aspect with a reason: the heat pump does transfer more energy to the water (about 2.6 kW) than it takes from the outlet (1.00 kW), as the data show; or energy cannot be created, because the first law (conservation of energy) holds for every system **1 point**

(d)(ii) **2 points**

For identifying the incorrect aspect: the conclusion that the meter must be wrong (or that the extra energy would have to be created) **1 point**

For explaining that the outlet is not the only source of energy for the refrigerant: it also takes in energy from the basement air, and the energy given to the water equals the electrical energy plus the energy from the air, so no energy is created and the meter reading is consistent with the data **1 point**

Example response: The heat pump really does give the water more energy than the outlet supplies, but energy is not created. The refrigerant also takes energy from the basement air in the outside coil. Over each trip around the loop its internal energy doesn't change, so what it gives to the water is the electrical work plus what it took from the air.

Trap: Agreeing with the student. A heat pump moves energy that is already in the basement air; it is not a heater that converts electrical energy alone.

(e) **1 point**

For selecting "Greater than" and correctly explaining that the heating element transfers energy to the water only at the rate it uses electrical energy, 1.00 kW, while the heat pump transfers energy to the water at about 2.6 kW (part (b)); since the same $mc\,\Delta T$ is needed, the element takes longer. Naming the energy from the air or the ratio of times (about 2.6) is not required (no credit for this part without the correct selection) **1 point**

Trap: "The same, because both use 1.00 kW." The water's warming rate depends on the rate of energy transfer to the water, not on the electrical power alone.

(f) **1 point**

For selecting "Greater than" and correctly explaining, with $Q = mc\,\Delta T$ or equivalent proportional reasoning, that for the same rate of energy transfer $\Delta T/\Delta t = (\text{rate})/(mc)$, so a smaller mass of water, which needs less energy per degree, warms faster (the factor of 1.5 is not required) (no credit for this part without the correct selection) **1 point**

Chapter 18c: Entropy and the second law

Question 18c.1 (Ball that stops bouncing, page 152) 10 points

(a) **1 point**

For selecting "The same as" and correctly explaining that no energy is transferred into or out of the system (it is isolated), so its total energy is constant (the student may also note that the gravitational potential energy of the ball–Earth system has become internal energy of the ball, carpet, air, and crate; this is not required) (no credit for this part without the correct selection) **1 point**

Trap: "Less than, because the ball stopped." The ball's mechanical energy decreased, but the system's internal energy increased by the same amount.

(b) **2 points**

For describing the energy at the start as concentrated (localized): it is associated with one object, the ball's height above the floor (and, while falling, the ball's motion as a whole, with all its particles moving together) **1 point**

For describing the energy at the end as spread out (dispersed) among the random motions of a very large number of particles in the ball, carpet, and air, and connecting this spreading to an increase in entropy: in an isolated system, energy spontaneously spreads out, and the entropy increases as it does **1 point**

Example response: At first the energy is all in one place: the height of the ball. Each bounce turns some of it into random jiggling of the molecules in the ball, carpet, and air. By the end, the energy is spread among a huge number of molecules instead of being concentrated in the ball. Energy that is concentrated tends to spread out, and the entropy of the isolated system increases as it does.

A correct response stated with counts of microstates is not what was asked; award the second point only if the spreading of energy is described. "Entropy is disorder" or "the ball got messier" without describing where the energy goes earns neither point.

(c)(i) **1 point**

For identifying a correct aspect with a reason: the reverse process would conserve energy, since the energy the carpet and air give up would equal the gravitational potential energy the ball–Earth system gains; or the carpet and air would have to cool for the ball to rise **1 point**

(c)(ii) **2 points**

For identifying the incorrect aspect, the claim that conserving energy is enough for the reverse process to happen, with a reason: conservation of energy is necessary but not sufficient, because another principle (the second law) decides which direction a process goes **1 point**

For explaining that the reverse process would require energy spread out among very many particles to gather on its own into the motion of one object, which would decrease the entropy of an isolated system; the second law of thermodynamics forbids this, so conservation of energy is necessary but not sufficient **1 point**

Example response: The student is right that the reverse would conserve energy. But energy conservation is not the only rule. Going backward, energy spread among countless molecules of carpet and air would have to collect itself into the ball's upward motion. That would lower the total entropy of an isolated system, which the second law says does not happen.

A response that the reverse is so unlikely that it would not be seen in any realistic time (far longer than the age of the universe) also earns the second point, if it names the entropy decrease (or the gathering of spread-out energy) as what would be required. Such a response must not agree that waiting would produce the reverse.

Trap: Agreeing with the student because energy is conserved. Conservation of energy says which processes are possible on energy grounds; the second law says which direction they go.

(d) **3 points**

For identifying the energy converted as the decrease in gravitational potential energy of the ball–Earth system, mgh (the ball starts and ends at rest) **1 point**

For setting the ball's share equal to the energy needed to warm it: $f\,mgh = mc\,\Delta T$ **1 point**

For $\Delta T = \dfrac{fgh}{c}$ **1 point**

Trap: Leaving m in the answer, or forgetting f and giving the ball all of the energy.

(e) **1 point**

For selecting "Decrease" and correctly explaining that the ball, carpet, air, and crate are no longer isolated: energy leaves them and spreads into the cooler room, so less energy is shared among their own particles and their entropy decreases (or: they return to their original temperature, the condition they were in before the energy spread into them, so their entropy goes back down); the second law requires only that the total entropy of the isolated system that includes the room not decrease (no credit for this part without the correct selection) **1 point**

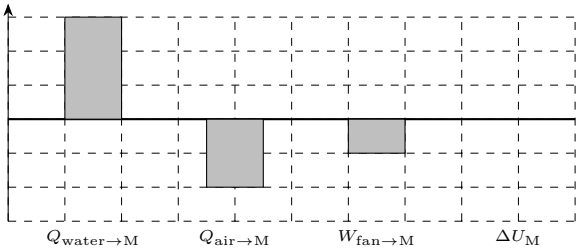
Trap: "Increase, because entropy always increases." The second law applies to an isolated system; the entropy of a system that loses energy by cooling can decrease.

Question 18c.2 (Thermoelectric fan on a mug, page 154) 12 points

(a) **2 points**

For $Q_{\text{water}\to\text{M}}$ positive, $Q_{\text{air}\to\text{M}}$ and $W_{\text{fan}\to\text{M}}$ both negative, and ΔU_{M} zero (no bar) **1 point**

For relative lengths consistent with $\Delta U_{\text{M}} = Q + W = 0$: the positive bar is as long as the two negative bars combined **1 point**



Courtesy chart. The split between the two negative bars is not graded; a real module delivers only a few percent of the energy it passes as electrical energy, so a much shorter W bar is realistic. Signs follow "energy transferred to the module"; a chart that uses the opposite convention consistently and says so earns both points.

(b) **3 points**

For the energy leaving the water, $mc\,(T_1 - T_2)$, all of which enters the module **1 point**

For the electrical energy delivered to the fan, $P\,\Delta t$, and for applying the first law to the module with $\Delta U_{\mathrm{M}} = 0$: energy in from the water equals energy out to the air plus electrical energy out to the fan **1 point**

For $Q_{\mathrm{M}\to\mathrm{air}} = mc\,(T_1 - T_2) - P\,\Delta t$ **1 point**

Trap: Writing $mc\,(T_2 - T_1)$, which is negative because the water cools, or adding $P\,\Delta t$ instead of subtracting it.

(c)(i) **1 point**

For identifying a correct aspect with a reason: energy that goes into the room air does not reach the fan (it is not converted to electrical energy); or energy is conserved, so energy that is not sent to the air must go somewhere else (no claim that it all reaches the fan) **1 point**

(c)(ii) **2 points**

For identifying the incorrect aspect: the conclusion that insulating the fins would let all of the energy from the water become electrical energy (or that the fan would keep running) **1 point**

For a correct explanation, either: (1) energy taken from the water is spread among the random motions of a huge number of molecules; converting all of it into the organized electrical energy for the fan, with none spreading out into a cooler place, would make the energy more concentrated and decrease the total entropy of an isolated system, which the second law forbids; or (2) with the fins insulated, energy entering the module has nowhere to go, so the finned face warms up to the temperature of the water, and then no energy passes through the module at all and the fan stops **1 point**

Example response: Energy sent into the room really doesn't run the fan. But all of the water's energy can't become fan energy. The water's energy is spread out among countless molecules; turning all of it into the fan's organized motion would concentrate energy and decrease the entropy of the isolated system, which can't happen. Some energy has to spread into a cooler place, here the room air.

Trap: Agreeing with the student because the first law balances. The first law allows it; the second law does not.

(d) **2 points**

For explaining that the water still has a great deal of internal energy at room temperature: its molecules are still moving randomly, and its temperature is far above absolute zero (U depends on the absolute temperature, which has dropped only slightly) **1 point**

For explaining that the fan stops because there is no longer a temperature difference between the water and the room air, so there is no net energy transfer through the module; energy spread evenly at one temperature is not available to run the fan **1 point**

Trap: Agreeing that "cooling to room temperature" means the water's energy is used up.

(e) **1 point**

For selecting "Decrease" and correctly explaining that energy leaves the water (by heating the module) so less energy is spread among its molecules and its entropy decreases (the student may also note that the water's volume and amount stay the same, or that the water is not isolated; neither is required) (no credit for this part without the correct selection) **1 point**

Trap: "Increase, because entropy always increases." The second law applies to an isolated system, not to the water alone.

(f) **1 point**

For selecting "Greater than" and correctly explaining that the closed room is an isolated system, so its total entropy increases as energy spreads from the hot water into the cooler room (the student may also note that the process is spontaneous or irreversible); for the total to increase, the entropy gained by the module, fan, and air must exceed the entropy lost by the water (no credit for this part without the correct selection) **1 point**

Chapter 18d: Supplement: microstates (beyond AP Physics 2)

Question 18d.1 (Two canisters and entropy, page 157) **10 points**

(a) **1 point**

For selecting "From X to Y" and correctly explaining that (1) the two canisters together are isolated, so their total entropy cannot decrease and increases in a spontaneous process, and (2) a given amount of energy raises the colder canister's entropy by more than it lowers the hotter canister's entropy, so energy moving from X to Y increases the total entropy while energy moving from Y to X would decrease it (no credit for this part without the correct selection) **1 point**

For (2), any of these reasons is enough: Y has less internal energy, and at lower internal energy each added bit of energy typically increases the number of accessible microstates by a larger factor; $\Delta S \approx \Delta U/T$, so the same energy gives a larger entropy change at the lower temperature; X's molecules have the greater average kinetic energy, so moving energy to Y spreads the total energy more evenly among all the molecules, which typically has more microstates. Writing $1/T = \Delta S/\Delta U$ or $S = k_B \ln W$ is not required. "Heat flows from hot to cold" by itself earns nothing, and "Y has less entropy, so it gains entropy" does not earn the point.

(b) **1 point**

For selecting "Less than" and correctly explaining that X loses internal energy while its volume and amount of gas stay fixed, so its entropy decreases (with less energy to share among its molecules, fewer microstates are typically available to it, or $\Delta S \approx Q/T$ with $Q < 0$); the second law requires only the total entropy of the isolated pair to increase, not the entropy of each part (no credit for this part without the correct selection) **1 point**

Students reason from small cartoons (a few oscillators sharing a few energy quanta), in which counts can behave differently from those for a real gas, and they are not expected to know about very large numbers of particles or about weighting microstates by probability. Do not require a claim that every count must fall; "typically fewer microstates" or "fewer ways to share the energy" with the decrease in X's energy as the cause is enough.

Trap: "Greater, because entropy always increases." The second law applies to the isolated pair, not to each canister.

(c) **2 points**

For reasoning that at a common temperature (equal internal energies, for identical canisters) a small amount of energy changes each canister's entropy by the same amount per unit energy, so moving a little energy either way raises one canister's entropy by as much as it lowers the other's (or: the number of microstates of the pair is typically greatest when the energy is shared equally) **1 point**

For concluding that no transfer in either direction can increase the total entropy, which is now at its greatest value, so the second law no longer favors either direction and there is no net flow (transfers in the two directions balance on average) **1 point**

Trap: "Because the entropy is now zero" or "because each canister has reached maximum entropy." Only the total entropy of the pair is at its maximum; each canister's entropy depends on its energy.

(d) **2 points**

For reasoning that the canisters are rigid and isolated from everything else, so no work is done and the energy X loses equals the energy Y gains; since the canisters are identical, they meet at the average temperature, $T_f = 350$ K **1 point**

For $Q = \frac{3}{2}nR\,\Delta T = \frac{3}{2}(0.20\text{ mol})(8.31\text{ J/(mol K)})(50\text{ K}) \approx 125$ J (accept 120–130 J) **1 point**

Writing $U = \frac{3}{2}nRT$ for each canister and using conservation of total internal energy is an equally good route to both points.

(e) **3 points**

For the change in entropy of X with the correct sign: $\Delta S_X \approx -125\text{ J}/375\text{ K} \approx -0.33$ J/K **1 point**

For the change in entropy of Y with the correct sign: $\Delta S_Y \approx +125\text{ J}/325\text{ K} \approx +0.38$ J/K **1 point**

For the total, $\Delta S \approx +0.05$ J/K, a positive value (accept 0.04–0.06 J/K, consistent with the values for X and Y) **1 point**

The exact values, $\frac{3}{2}nR\ln(T_f/T_i)$, are -0.333 and $+0.385$ J/K, total $+0.051$ J/K; the approximation is not the source of the positive total.

Trap: Using the same temperature for both canisters, which gives a total of zero.

(f) **1 point**

For selecting "Less than" and correctly explaining that the same energy (125 J, since the temperature difference and heat capacity are unchanged) is transferred, and the total is about $Q\,(1/T_{Y,\text{avg}} - 1/T_{X,\text{avg}}) = Q\,(T_{X,\text{avg}} - T_{Y,\text{avg}})/(T_{X,\text{avg}}T_{Y,\text{avg}})$: the numerator is unchanged (50 K) while the product of the temperatures is larger, so Y's gain exceeds X's loss by less (about +0.03 J/K; a calculation as in (e) also earns the point) (no credit for this part without the correct selection) **1 point**

Trap: "Equal to, because the temperature difference is the same." The total depends on $1/T_C - 1/T_H$ (roughly), not on $T_H - T_C$.

Question 18d.2 (Two tiny solids sharing quanta, page 159) 10 points

(a) **1 point**

For a complete list of the six microstates with no repeats, giving $W = 6$: $(2,0,0)$, $(0,2,0)$, $(0,0,2)$, $(1,1,0)$, $(1,0,1)$, $(0,1,1)$ **1 point**

Trap: Counting only the two patterns (both quanta in one oscillator; one quantum in each of two oscillators) and answering 2, or treating the two quanta as distinguishable and answering 9. The quanta are identical, so a microstate is fixed by how many quanta each oscillator holds.

(b) **1 point**

For $W = W_A W_B = 15 \times 1 = 15$, so $S = k_B \ln 15 \approx 2.7\,k_B$ (accept $k_B \ln 15 + k_B \ln 1$) **1 point**

(c) **2 points**

For the rows $W_A = 15, 10, 6, 3, 1$ and $W_B = 1, 3, 6, 10, 15$ **1 point**

For the products $15, 30, 36, 30, 15$ (consistent with the student's W_A and W_B) **1 point**

The products total 126, the number of microstates of the pair after contact. The total is not required in (c) but is needed in (e).

(d) **1 point**

For $q_A = 2$, $q_B = 2$, with the correct reason: every microstate of the pair is equally likely, and this split has the most microstates (36 of the 126), more than any other split (no credit for this part without the correct split) **1 point**

The pair is still found in other splits most of the time (90 of 126 microstates); "most likely" compares splits, not this split with all the others together.

Trap: "All 4 in B, because energy flows from A to B." Energy does flow from A to B on average, but only until it is shared about equally.

(e) **2 points**

For using the total number of microstates after contact, $W = 15 + 30 + 36 + 30 + 15 = 126$ (all of which are accessible), not the 36 of the most likely split **1 point**

For $\Delta S = k_B \ln 126 - k_B \ln 15 = k_B \ln 8.4 \approx 2.1\,k_B$ (about 2.9×10^{-23} J/K) **1 point**

A student who uses $W = 36$ ($\Delta S = k_B \ln 2.4 \approx 0.88\,k_B$) earns the second point only, if the arithmetic is otherwise correct. For this tiny pair the most likely split holds only 36 of the 126 microstates, so it cannot stand in for the total.

Trap: Subtracting microstate counts $(126 - 15)$ instead of taking the logarithm of the ratio.

(f) **1 point**

For selecting "Solid A" and correctly explaining that one more quantum raises the entropy of A only from $k_B \ln 15$ to $k_B \ln 21$ ($\Delta S \approx 0.34\,k_B$) but raises the entropy of B from $k_B \ln 1$ to $k_B \ln 3$ ($\Delta S \approx 1.1\,k_B$); a smaller $\Delta S/\Delta U$ means a smaller $1/T$, so A is hotter (comparing A losing one quantum, $\ln(15/10) \approx 0.41\,k_B$, with B gaining one gives the same ranking) (no credit for this part without the correct selection) **1 point**

Trap: "Neither, because they are the same kind of solid." Temperature depends on how much energy each solid holds, not on what it is made of.

(g) **2 points**

For explaining that the entropy of the pair is set by the number of accessible microstates (126 after contact), which does not change when the pair happens to be in one particular microstate; the pair visiting a microstate with all quanta in A is part of exploring all 126, not a decrease in entropy	**1 point**
For explaining from the counts that the return is expected: the all-in-A split has 15 of the 126 equally likely microstates (about 12%), so the pair is found there about one look in eight, while it spends most of its time (111 of 126) in other splits; the second law describes the most likely behavior, not a ban on any microstate	**1 point**

A student who instead treats the entropy as $k_B \ln$ of the current split's microstates (so it does dip briefly) earns the first point by explaining the dip as the pair visiting equally likely microstates, with a count or probability cited, so the dip is predicted by the counting and is not a violation; the second point is earned as above. Remarks that such returns become less likely the more oscillators there are are welcome but not required.

Trap: Agreeing with the student, or saying the second law makes the return impossible for the tiny pair.

Notes: provenance records

Prepared with assistance from Claude, an AI model made by Anthropic. Each record lists separate choices, made and written down before the question's wording: the governing and constitutive relationships, the branch of the author's production schema (handout T.0.2) that the question turns on, the object model and representation, the format (handout T.1), and the trap. A script compares every record with every other and flags repeated object models and repeated pairings of relationship and branch. No published question was open while any question was written. Questions marked "bank" are copied from the author's own CC0 problem bank. Numerical answers and experimental data were checked by script. This describes the process used; it is not legal advice.

Q	Relationships	Schema branch	Object model	Trap; source
AP Physics 1				
1.1	kinematics (displacement = area under v-t); avg v = (vi+vf)/2 for UAM	OS.T.2 limiting cases of phase durations	programmable toy car with two phases	larger v1 so smaller acceleration; bank (2021 EoY packet, 1.1.3)
1.2	kinematics (UAM); avg v = (vi+vf)/2	OS.T.2 equal time intervals at half speed	braking car on highway	half speed at half distance; bank (2021 EoY packet, 1.1.2)
1.3	average velocity = displacement / time; x-t graph slope	OS.T.2 interval set by A's finish	two cars racing to a finish line	same track so same average velocity; bank (2021 EoY packet, 1.1.1)
2.1	2-d kinematics (vertical motion sets time); y = vy0 t - g t^2/2	OS.T.2 split flight at launch height	cannonball from cliff	time proportional to launch speed; bank (2021 EoY packet, 1.1.4)
2.2	2-d kinematics (released object keeps carrier's velocity); x = w t; y = H + u t - g t^2/2	OS.T.2 initial vertical velocity nonzero at release	seed pod released from a climbing survey drone	'dropped' means starts from rest; new
3.1	N2L (equilibrium); static friction ≤ muS N; magnetic pull given	1 snapshot; inequality fs≤muS N	fridge magnet + calendar	fs = muS N misread as equality; new
3.2	N2L (system); weight	system choice: inertia of whole system	block on table + hanging mass	only block has inertia; bank (2021 EoY packet, 1.2.3)
3.3	N2L; weight + downward normal force	OS.T.1 two forces, only one halved	block pushed down by a moving panel	acceleration proportional to one force; bank (2021 EoY packet, 1.2.1)
3.4	N2L (hanging and horizontal); F = m_g g; F = m_i a	two measurements, gravity-dependent and not	cart, force probe, motion detector	one measurement called the mass; bank (2021 EoY packet, 1.2.2)
3.5	N2L (system) + kinematics; f = muK N; v^2 = v0^2 + 2a dx	OS.T.2 reaches smooth region before stopping	block skidding on half-rough table with hanging mass	friction beats hanging weight so it stops; bank (2021 EoY packet, 1.2.5)
3.6	N2L (system and sphere); f = muK N; N includes string pull	OS.S system choice; normal force carries sphere weight	block on two planks with a sphere hanging ahead through the gap	lighter block less friction smaller a; bank (2021 EoY packet, 1.2.6; Ch 3 Quiz 3)
3.7	N2L (verge of slipping); fs ≤ muS N	Si.Vrg.Sl verge of slipping	wooden block on wooden board, claimed friction ceiling	no plateau seen means consistent; bank (2021 EoY packet, 1.2.7)
3.8	N2L along incline; mg sin; kinetic friction opposes motion	OS.T.2 up vs down	block sliding up or down a ramp	gravity helps going down; bank (2021 EoY packet, 1.2.8)
3.9	N2L (2-d); N cos = mg; N sin = ma	tan theta = a/g nonproportional	box held on an accelerating wedge	angle proportional to acceleration; bank (2021 EoY packet, 1.2.9)
3.10	N2L; slope of v-t = a; mg - Fd = ma	constant slope means constant drag	object falling through fluid, curved then straight v-t	concave portion supports claim; bank (2021 EoY packet, 1.2.10)
3.11	N2L (2-d) for ball and boxcar; kinetic friction opposes motion	string perpendicular to ramp when a = g sin	ball hanging in a boxcar on a ramp	string hangs vertical; bank (Boxcar handout)
4.1	N2L (circular); Fg = G m1 m2 / r^2; a = v^2/r	OS.T.1 orbit radius vs separation	binary pair of equal stars	mass cancels as for satellite; bank (2021 EoY packet, 1.3.1)
4.2	N2L (circular) for each moon; Fg = GMm/R^2; v = 2 pi R/T	linearize T^2 vs R^3; enclosed mass differs	four moons of a new planet	line not forced through origin; bank (2021 EoY packet, 1.3.2)
5.1	impulse-momentum; p=mv	2 snapshots; external impulse nonzero	cart + spring bumper	speeds vs velocities; new
5.2	momentum + energy; spring U	2 snapshots; momentum toggled off by glue	two blocks + spring	same energy so same compression; new

Q	Relationships	Schema branch	Object model	Trap; source
5.3	momentum conservation; p = mv vs mistaken mv^2/2	OS.S.ChCons.Imp; wrong conserved quantity	identical sticky carts; misremembered formula	same answer either way; bank (2021 EoY packet, 1.4.2)
5.4	impulse-momentum (horizontal); pulley force on string	OS.S external force from pulley	block on table, string over pulley, hanging mass	tension supplies the impulse; bank (2021 EoY packet, 1.4.4)
6.1	COM of isolated system stays at rest; x_cm = sum m x / sum m	OS.S.IntDOFS internal force cannot move COM	two carts held by thread with repelling magnets, thread burned	COM drifts toward heavier cart; new
6.2	COM velocity constant with zero external force; x_cm = (m1 x1 + m2 x2)/(m1+m2)	OS.S choose two-puck system through collision	air-hockey pucks filmed from above, one stationary	COM changes during the collision; new
7.1	work-energy; W = F dl; K = mv^2/2	OS.S.ChCons.WChInt same work, different mass	blocks pushed on frictionless track	same work so same speed; bank (2021 EoY packet, 1.5.2)
7.2	conservation of mechanical energy; UG = mgh; K = mv^2/2; time = distance/speed	OS.T.2 speed set by final height; contact constraint	toys inserted in slots under a sliding ball	tallest toy without checking contact; bank (2021 EoY packet, 1.5.4)
7.3	energy then momentum; v = sqrt(2gH); perfectly inelastic	OS.T.2 two stages, different conserved quantity	block down ramp hits and sticks to block on floor	conserve KE through collision; bank (2021 EoY packet, 1.5.5)
7.4	momentum + energy; common velocity at max height	OS.T.2 shared motion at max height	block sliding up a free wedge, masses swapped	heavier block climbs higher; bank (2021 EoY packet, 1.5.6)
7.5	energy; spring U; UG	OS.S system choice (ball alone vs ball+spring)	piston-spring ball launcher, horizontal vs vertical	more compression more ball energy; bank (2021 EoY packet, 1.5.7)
8a.1	N2L rotation (static equilibrium); tau = r_perp F	OS.A choose axis through intersection of lines of action	rod resting in hemispherical bowl	keep figure's arrangement; bank (2021 EoY packet, 1.6.1)
8a.2	N2L rotation (static) + N2L; tau = r_perp F	OS.A axis at one support; one support pushes down	wall shelf held by an upper and a lower peg with a cat at the free end	both pegs push up; new
8b.1	N2L rotation; tau=r_perp F; I=sum m r^2	1 snapshot; torque zero when line of action through axis	disks with clay lumps	force size read as torque; new
8b.2	work-energy (rotational) and N2L rotation; W = F L; K = 1/2 I w^2; tau = rF; L = r dtheta	2 snapshots; work depends on string length, not spindle radius	pull-cord top launcher with interchangeable spindles	bigger torque so bigger final spin; new
8b.3	angular momentum conservation; N2L rotation; I = MR^2/2; tau = mu N R	scaling cancels in both torque and inertia	spinning disk on rubber feet atop a second disk	heavier presses harder so longer marks; bank (2021 EoY packet, 1.7.2)
8b.4	angular momentum + energy; K = L^2/2I; common omega at max height	OS.T.2 same L, different K	ball in groove of lazy Susan with ramp	same L so same height; bank (2021 EoY packet, 1.7.3)
10.1	SHM (half cycles); T = 2 pi sqrt(m/k); energy kA^2/2	OS/Si.SHM recognize portion of SHM	unattached block between two springs	attached-spring period; equal amplitudes; bank (2021 EoY packet, 1.8.2)
10.2	SHM (pendulum portion) + kinematics; T = 2 pi sqrt(R/g); v = sqrt(2gh)	OS/Si.SHM portion plus constant-speed segment	skate ramp with two quarter arcs	pendulum period independent of everything; bank (2021 EoY packet, 1.8.4)
10.3	N2L; SHM; f = b x m g acts like spring	OS/Si.SHM hidden SHM	block sliding onto floor with friction growing with distance	acceleration linear in time; bank (2021 EoY packet, 1.8.1)
10.4	SHM (pendulum quarter periods); T = 2 pi sqrt(l/g)	OS/Si.SHM two portions of different length	pendulum string catching on a lower rod	longer apparatus longer time; bank (2021 EoY packet, 1.8.3)
11a.1	N2L for a system (scale normal force vs total weight); buoyant force rho V g; COM motion	system choice: tank+water+float; just before vs just after	submerged float tethered in a fish tank on a bathroom scale	'nothing added or removed, so no change'; new
11a.2	N2L (equilibrium) for floating can; FB = rho V g; dV = A d	OS.T.2 same added weight	two open aluminum cans of different diameters floating	same mass so same depth; bank (2021 EoY packet, AP Physics 2 1.1.1)
11b.1	continuity (volume flow rate); A v = const; free fall speeds up	OS.T.2 two heights in one steady stream	thin falling stream from a kitchen faucet	pressure squeezes stream; new
11b.2	continuity; Q = A v; A = pi d^2/4	OS.T.2 fixed volume rate through different needles	syringe pump with interchangeable needles	speed proportional to 1/d instead of 1/d^2; new

Q	Relationships	Schema branch	Object model	Trap; source
11c.1	Bernoulli + hydrostatic pressure; P + rho g y + rho v^2/2 = const	OS.T.2 surface and outlet; loop height drops out	aquarium siphon hose with adjustable loop	higher loop slower flow; new
11c.2	hydrostatic pressure + Bernoulli + projectile; P = P0 + rho g d; v = sqrt(2gd)	OS.T.2 two holes; range needs fall time	soda bottle with two pin holes above a counter	higher pressure so farther jet; new
AP Physics 2				
7.1	energy conservation on U(x); F = -dU/dx	turning points and force from slope	magnet cart on track with fixed magnets	zero U slope means zero speed; new
7.2	energy conservation on U(r); F = -dU/dr; asymmetric well	average separation rises with energy	two bonded atoms in a solid	symmetric vibration so no expansion; new
11.1	standing waves on string; v = sqrt(T/mu); f1 = v/2L	linearize f^2 vs T	guitar string tensioned via luggage scale	plot f vs T; new
11.2	standing sound waves in closed tube; f = v/4L for air column	air-column length not water depth	glass bottles with water, blown across top	double water double pitch; new
11.3	equal travel times: d/v_sound = L/(c/n); v = c/n in fiber; total internal reflection at core-cladding boundary	one-way vs round trip (factor of 2 cancels); propagation delay vs bandwidth	two musicians linked by an ideal optical-fiber route	treating bandwidth as signal speed; new
11.4	superposition of waves from two synchronized sources; in phase when path difference = n lambda, out of phase when (n + 1/2) lambda; v = f lambda; diffraction most pronounced when lambda is comparable to or larger than the opening	every point on the line beyond one source has the same path difference (the source separation); halving lambda turns a quiet spot loud; removing one source makes a quiet spot louder	two stage loudspeakers driven by one tone generator in an empty theater, side door to a hallway	closer to a speaker is always louder; fewer speakers is always quieter; quiet spots stay put when the pitch changes; new
12a.1	dU = q dV; energy conservation; |E| = |dV|/dl; E points downhill	path independence; sign of charge	lab map of equipotential lines (unknown source)	longer path more work; electron moves downhill; new
12a.2	dV = -E dr (area under E-r); E = kQ/r^2 outside conductor	which region contributes most area	charged conducting sphere with three equal radial regions	equal widths so equal contributions; bank (2021 EoY packet, 3.1.2)
12b.1	loop/junction rules; Q=C dV; Ohm	just after vs long after	RC divider	capacitor as break at t=0; new
12b.2	R = rho l / A; fixed volume: A = Vol/l	hidden coupling between controlled and changed variable	cylinders rolled from one lump of conductive dough	R proportional to l when A also changes; new
13.1	N2L (circular) with magnetic force + energy; F = qvB; qV = mv^2/2	compare species through same V and B; period independent of speed	proton therapy bending magnet with protons and alpha particles	larger charge bends more; new
13.2	magnetic force between currents + N3L; B = mu0 I / 2 pi r; F = I L B	N3L pair equal despite unequal currents	two single-conductor DC cables on a workshop bench carrying unequal currents	larger current exerts larger force; new
14.1	magnetic force on moving charge (motional emf); F = qvB; flux symmetric	relative motion vs changing field	copper disk and coaxial cylindrical magnet (Faraday disk)	spinning magnet induces emf; bank (2021 EoY packet, AP Physics 2 1.2.1); the cases follow Faraday, Experimental Researches in Electricity, paragraphs 88 (1831) and 218 (1832), public domain
14.2	motional emf + Ohm; emf = B v w; R prop. perimeter; P = emf^2/R	width perpendicular to velocity matters	loops bent from one wire stock pulled into field region	area or perimeter instead of leading width; new
14.3	Faraday's law (motional emf) + Newton's second law + energy conservation; emf = B l v; I = emf/R; F = I l B; P = I^2 R	terminal speed set by force balance; R doubled; open circuit (emf but no current)	horizontal rod sliding down vertical rails joined by a resistor in a uniform field	larger resistance slows the rod more; an emf alone retards the rod; new
15.1	single-slit diffraction; a sin theta = m lambda; small angle	linearize spacing vs screen distance	laser through a human hair onto a wall	thicker hair wider pattern; new

Q	Relationships	Schema branch	Object model	Trap; source
15.2	thin-film interference; phase flip at higher-n reflection; 2 n t	number of phase flips (coating index between air and glass)	clear coating on a glass slide vs soap film	thin film always dark; new
16.1	thin lens; 1/f = 1/so + 1/si; rays	every part of lens forms whole image	magnifying glass projecting a window onto a wall	covering half lens removes half image; new
16.2	Snell's law; n1 sin1 = n2 sin2; small angle	refraction away from normal into air	coin at bottom of a mug of tea	coin appears deeper; new
16.3	law of reflection; mirror equation; 1/do + 1/di = 1/f; M = -di/do; plane-mirror image as far behind as object in front	object inside focal point (virtual, upright, magnified) vs convex mirror (diminished); plane mirror size independent of distance	person at a plane wall mirror and a concave makeup mirror	mirror must be as tall as the person; moving away from a mirror always shrinks the image; new
16.4	rectilinear propagation of light (ray model); similar triangles; brightness proportional to visible source width (stated model)	extended vs. point source; obstacle narrower than source so the full shadow ends before the wall	glowing frosted window, tall thin board, far wall of a room	point-source thinking: sharp shadow; narrower source gives smaller shadow; new
17a.1	energy conservation for photons; E = h f; e V	linearize turn-on voltage vs frequency	set of colored LEDs on a variable supply	intensity sets photon energy; new
17a.2	energy conservation for atomic transitions; E photon = level difference	cold gas absorbs only from ground state	hypothetical three-level gas lamp and cool cloud	absorption at every emission line; new
17a.3	energy conservation for atomic transitions (photon energy = level difference); Bohr hydrogen-like E_n = -13.6 eV Z^2/n^2, r_n = 0.0529 nm n^2/Z; E = hf; net enclosed charge of ion core	far-out electron sees Z_eff = 1; at low n the electron overlaps the core and Z_eff ¿ 1	rubidium atom with outer electron excited to n near 50	Z = 37 (nuclear charge); transition energy scales as 1/n^2; new
17b.1	conservation of momentum and energy in a decay from rest; E = (delta m) c^2 with 1 u = 931.5 MeV/c^2; K = p^2/2m; conservation of charge and nucleon number	equal and opposite momenta give the lighter fragment most of the kinetic energy	Po-210 nucleus at rest decaying to Pb-206 + alpha	energy split equally, or all of Q given to the alpha; new
17b.2	radioactive decay as a random process with constant half-life; N = N0 (1/2)^(t/T); net rate = measured rate - background	background subtraction is needed before reading the half-life	medical Tc-99m vial on a counter with background	half-life read from raw counts (about 7.5 h); new
18a.1	kinetic theory; avg KE = 3kT/2; P = N k T / V	mixed gases reach one temperature	helium and xenon sharing a sealed bulb	heavier atoms more energy; new
18a.2	ideal gas law; V prop. T at fixed P	extrapolate to V = 0	greased glass syringe in water baths	plot in Celsius through origin; new
18a.3	kinetic theory (equal average KE at a common temperature); avg KE = 3kT/2 = m v_rms^2/2; area under the distribution = fraction of atoms	two gases at one temperature: peak position set by mass, peak height set by both number and spread	sealed container of 60% helium and 40% krypton at 300 K	taller peak means more atoms; new
18b.1	first law of thermo; PV=nRT; U=3/2 nRT	2 snapshots; path dependence of Q	gas in cylinder (isobaric vs isothermal)	same final volume so same heat; new
18b.3	first law of thermo over a cycle; W = area; dU = 0 over cycle	2 snapshots same state; Q net = W	rectangular P-V cycle of a demo engine	same state so net heat zero; new
18b.4	first law over whole cycles of a heat pump (refrigerant returns to its state: dU = 0); Q = m c dT for the water; electrical energy = P t; rate from slope of T-t	energy delivered to the water exceeds the electrical energy because the rest is drawn from the cooler room air	heat-pump water heater in a basement warming an insulated tank	a device cannot deliver more heat than the electricity it uses; new

Q	Relationships	Schema branch	Object model	Trap; source
18c.1	second law (qualitative): entropy of an isolated system increases as localized energy spreads; Q = m c dT; mechanical energy mgh	organized energy of one object becomes random energy shared by many particles; reverse process conserves energy but is forbidden	rubber ball dropped onto a carpet in a sealed insulated crate until it rests	energy conservation alone allows the ball to bounce back up; new
18c.2	first law for a device in steady operation + second law (qualitative): some energy must leave to the cold side; Q = m c dT; electrical energy = P t	energy spread uniformly at one temperature is unavailable for work; entropy of a closed part can fall	thermoelectric module on a mug of hot water driving a small fan	water at room temperature has run out of internal energy; insulate the cold side to convert all of it; new
18d.1	second law (isolated pair); dS = dU/T; S = kB ln W; U = 3/2 nRT	energy moves toward the larger entropy gain per unit energy; part entropy may fall	two identical insulated canisters of gas brought into contact	each canister's entropy increases; adapted from the user's two-canister fragment
18d.2	second law (statistical): all accessible microstates equally likely; S = kB ln W; ways to share q quanta among N oscillators; 1/T = dS/dU (given)	most probable split of quanta between two equal small solids; fluctuation back to the initial split is likely only for tiny systems	two three-oscillator Einstein-solid cartoons sharing four quanta	seeing the initial split again means the second law failed; new

Changelog

Question numbers are permanent: a new question gets the next number in its chapter, and no question is renumbered. Corrections to posted questions are listed here and appear in the next edition.

- **Phase 1.** First posted edition: 67 questions (45 AP Physics 1, 22 AP Physics 2); every chapter has at least two.
- **Phase 1.** Imported from the 2021 EoY packet: 1.1.1–1.1.4, 1.2.1, 1.2.2, 1.2.5–1.2.10, 1.3.1, 1.3.2, 1.4.2, 1.4.4, 1.5.2, 1.5.4–1.5.7, 1.6.1, 1.7.2, 1.7.3, 1.8.1–1.8.4, AP2 1.1.1, AP2 1.2.1, 3.1.2 (1.2.3 was already in the bank; 1.2.6 is the same question as Ch 3 Quiz 3), and the Boxcar handout. Keys were written where the sources had none.
- **Phase 1.** Changes to imported questions: 1.1.4 now says the launch direction is unchanged and air resistance is negligible. 1.1.2 is restructured from a single paragraph-length response into three parts: a qualitative comparison (greater, less, or the same), a velocity–time sketch, and an algebraic derivation of the ratio. 1.5.4 states that the ball has enough speed to reach the end and gives the order of the heights. 1.8.4 states that the ball slides without rolling; part (b) now asks for one consistent and one inconsistent feature of the scatterplot (the original could be argued either way). 1.6.1's figure is redrawn so that point P is clearly off line ℓ. 1.8.2(b) no longer implies which spring is stiffer, and 1.8.2 is rescored as a 7-point, 13-minute question (it was 12 points). 3.1.2 sketches are redrawn. 1.7.2 (stacked disks) is redrawn in correct perspective with the three feet 120° apart, plus a top view showing the feet. 1.1.1 adds a position–time sketch. 1.2.1 asks for a paragraph-length response and states that air resistance is negligible. 1.2.6 adds a top view (block across the slit; string and sphere seen through it) and a direction-of-motion arrow and says the string passes between the planks; the measured angles are raised by 30° (now 51–63°), because the original angles implied kinetic friction coefficients of about 1.5–2.6. 1.2.7: the claim now names its limiting value (8.0 N); as first written, the data could not contradict it, because the leveling off might happen above the largest normal force tested. 1.2.10 adds Yes/No selection lines and states that the fluid is uniform and the buoyant force is negligible. 1.5.7(a) now asks about the ball while it is held at rest. 1.5.5: the zero of gravitational potential energy is now specified (part (a) had no definite answer without it). 1.5.6 states that the initial speed is the same in both experiments and that the block does not reach the top. 1.7.3 (Lazy Susan) states that the ramp keeps the ball at the same distance from the axle and that the ball never reaches the cliff, keeps a perspective view and adds a top view drawn from the same 3-D model (the earlier top view put the ball on the wrong side of the ramp); the perspective view is drawn with hidden lines removed by ray tests (an earlier draft showed the groove through the opaque ramp), and adds part (c), a derivation of the first height. 1.8.1 adds part (b), a derivation of the stopping time. 1.8.3 states that the string length stays the same when the lower rod is moved and that the rod's small horizontal offset is neglected, redraws the figure with the rods seen end-on in front of the board, and adds part (b), a derivation of the time. The Boxcar handout's multiple-choice and explanation pairs are combined into two parts. The 1.3.2 key now notes that moons A and B agree with each other, as do C and D.
- **Phase 1.** Scoring rule for selections, applied to every question. A brief part with selection blanks followed by a brief explanation is worth exactly 1 point; the selection is a gate, and with the correct selection the point still requires a correct explanation (a good-faith attempt earns nothing). A paragraph-length part with selection blanks is not gated: the blanks are a courtesy and earn nothing; the points come from the paragraph's reasoning. Rescoring changed the following imported questions (totals and times unchanged). 1.1.1(a) is now paragraph-length. 1.1.2(a) is now paragraph-length (3 points) and (c) is 2 points. 1.1.3(a)(i), (a)(ii), and (d) are gated 1-point parts and (e) is a 4-point paragraph. 1.1.4 gains part (b), a gated selection on the maximum height. 1.2.1, 1.2.9, 1.2.10, and 1.4.2 each become a 6-point paragraph (a) plus a gated 1-point part (b);

1.5.6 becomes a gated 1-point part (a) followed by a 6-point paragraph (b). 1.2.5 is split into three calculations (deceleration on the rough portion, speed at the smooth portion, acceleration there) and a gated graph choice. 1.2.6(a)(i) is paragraph-length; (a)(ii) and (c)(i) are gated; new (c)(ii) asks for the kinetic friction coefficients of two groups; the old (c)(ii) is now (c)(iii) (2 points). 1.2.7 gains (f), the static friction coefficient from the data, and its design parts accept a tilting-board method. 1.2.8, 1.5.7, and the Boxcar handout: (a) is gated, (b) is paragraph-length, and a new derivation or calculation part (c) is added. 1.3.1(a) is paragraph-length. 1.5.4(a)(i) and (a)(ii) are paragraph-length, (b) and (c) are gated, and new (e) derives a speed. 1.5.5 is reorganized: (a) gated, (b) a derivation of the kinetic energy after the collision, (c) a gated comparison, (d) a derivation of the energy converted to internal energy. 1.6.1's selection point is replaced by a second reasoning point. 1.7.2(a) is gated and new (c) derives the energy converted to internal energy. 1.7.3(a) is gated and new (d) derives the ratio of the two heights. 1.8.1(a), 1.8.3(a), and 1.8.4(a) are paragraph-length. AP Physics 2 1.1.1 gains part (b), a gated selection on floating in salt water. 3.1.2(a) is paragraph-length, (b) and (c) are gated, and new (d) and (e) ask for the particle's kinetic energy at two points and a sketch of kinetic energy versus distance.

- **Phase 1.** AP Physics 2 Chapter 18b gains a question built from the user's two-canister fragment: gated entropy explanations of the direction of energy flow and of the hotter canister's entropy change, an explanation of why the flow stops at a common temperature, and calculations of the energy transferred and the entropy changes.
- **Phase 1.** AP Physics 2 1.1.1's figure is redrawn from a 3-D model, with the cans open at the top, the bottoms meeting the sides with vertical tangents, and the water drawn as a tank seen through its front.
- **Phase 1.** Every page carries the davidliao.com wordmark.
- **Phase 1.** Paragraph-length responses now appear only in the short (7-point) AP Physics 1 questions; every 12-point AP Physics 1 question and every AP Physics 2 question uses modest-length explanation parts instead. Imported questions affected: 1.1.3(e) is a 1-point gated selection and a new (f) derives the second-phase acceleration. 1.2.6(a) opens with a 2-point explanation of why the normal force on the block is $(m + M)g$, followed by gated selections for the acceleration and the angle; (c) now starts with the kinetic friction calculation, asks (gated) whether the change exceeds the measurement uncertainty, then asks the sabotage question, and ends with a 1-point alternative explanation tied to the data. 1.5.4(a) is three short parts (energy explanation; why Toy A is required; gated slot selection), and a new (e) asks for the difference in travel times; the former (e) is (f). 1.7.3 gains (b), why the disk starts turning counterclockwise; the paragraph is replaced by (c), how the kinetic energies at maximum height compare, (d), a gated comparison of the initial kinetic energies, and (e), a gated comparison of the heights; the derivations are (f) and (g). 3.1.2(a) is a 2-point explanation and a gated selection, and (d) also asks for the fraction of the kinetic energy gained in region I.
- **Phase 1.** AP Physics 2 Chapter 17 is split into 17a Quantum physics and 17b Subatomic physics (special relativity is not part of the course except mass–energy equivalence). Former 17.1 and 17.2 are now 17a.1 and 17a.2.
- **Phase 1.** New AP Physics 2 questions: 11.3 (playing music over optical fiber), 14.3 (rod falling on vertical rails), 16.3 (bathroom mirrors), 16.4 (window, board, and shadow), 17a.3 (Rydberg atoms), 17b.1 (alpha decay of polonium-210), 17b.2 (technetium-99m vial), 18a.3 (two-peaked speed distribution).
- **Phase 1.** The footer on landscape contents pages now runs along the bottom edge of the page as it is read.

- **Phase 1.** Changes to AP Physics 2 1.2.1 (copper disk and magnet): it says the magnet is cylindrical and magnetized along its length and asks about the potential difference between the dots (an emf belongs to a closed path); the magnet and disk are redrawn from a 3-D model; it now begins with the spinning-disk derivations (force on a free electron; magnitude and direction of the balancing field) and gated parts on doubling the angular speed, on which dot is at higher potential, and on the inner and outer halves of the span, and ends with the four-scenario table and gated explanations for a spinning magnet with a still disk and for both spinning. Its provenance now cites Faraday's own experiments (paragraphs 88 and 218 of Experimental Researches in Electricity).

- **Phase 1.** AP Physics 2 Chapter 18b is split three ways, following the author's handouts 18.1 and 18.2 and the 2024 course description (heating includes $Q = mc\,\Delta T$; the second law is treated only qualitatively): 18b Thermodynamics: first law (energy conservation, including heat engines and heat pumps, with no entropy); 18c Entropy and the second law (qualitative: concentrated energy spreads out; no microstates); 18d Supplement: microstates (beyond AP Physics 2). Question numbers are kept: 18b.1 and 18b.3 stay in 18b. Former 18b.2 (two canisters and entropy) is now 18d.1, and the number 18b.2 is retired, not reused. 18d.1 now opens with a note that it goes beyond the course. New questions: 18b.4 (heat-pump water heater), 18c.1 (ball that stops bouncing), 18c.2 (thermoelectric fan on a mug), 18d.2 (two tiny solids sharing quanta).

- **Phase 1.** AP Physics 2 11.2 (blowing across jars): the figure for sketching in part (a)(i) is now an enlarged copy of jar B with the same proportions (it had been drawn wider than jar B), and the scoring guidelines add a courtesy figure for that sketch. New question 11.4 (two speakers in a theater): path-length interference on a grid map, a student's claim about unplugging one speaker, and diffraction of sound through a doorway compared with light.

- **Phase 1.** AP Physics 2 11.4(f)–(g): speaker S_2 is now unplugged and the frequency reset before the doorway parts (the two-speaker interference pattern would otherwise change the sound reaching the doorway), the door is about $8d$ wide instead of $2d$ (with the wavelength $4d$ already larger than a $2d$ door, lowering the frequency changed the spread very little), the hallway walls absorb sound, and (f) asks whether the sound spreads over a wider, narrower, or the same range of directions instead of asking about an "amount" of sound.

- **Phase 1.** AP Physics 1 5.4 (horizontal momentum and a pulley): the figure is redrawn so the pulley clears the edge of the table, held by a bracket attached to the table's side (it had been drawn overlapping the table, where it could not turn). The same fix is made to the pulleys in 3.2 (block and hanging mass) and 3.5 (half-rough table).

- **Phase 1.** Fixed surfaces (floors, tabletops, walls) are now drawn in one style in every figure: a thick contact line with hatching on the solid side, and a drawn end edge where a table stops. Figures changed: AP Physics 1 3.5 (the table now has an end edge; the frictionless section keeps a lighter hatch), 3.2 (line weights match the other figures), and, with no visible change beyond hatch depth, 3.1, 5.1, 5.4, 7.3, 7.4, 10.3, 10.1, 8a.2.